Gluonic Poles T-odd PDFs and FFs

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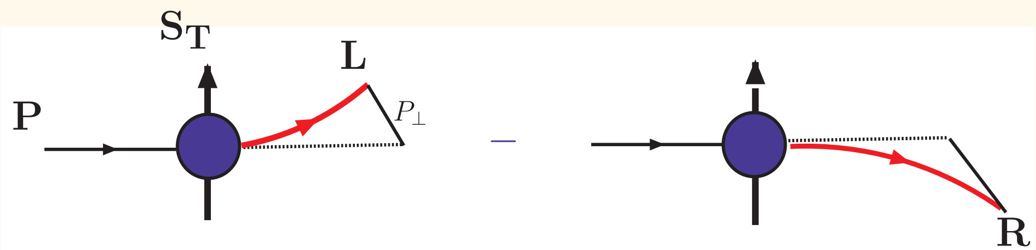
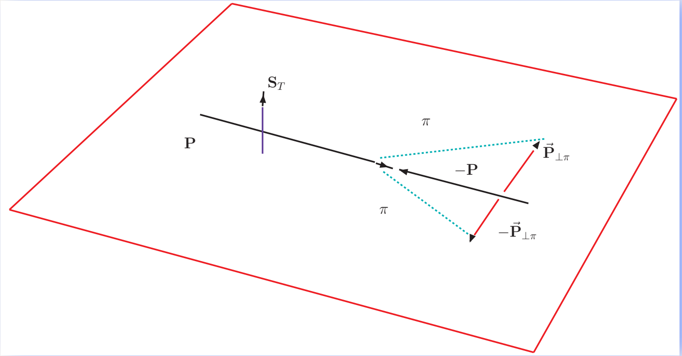
Based on PRD 77 114026 (08) With Piet Mulders & Asmita Mukherjee
and also Marc Schlegel

OUTLINE

- **Transverse *Single* Spin effects Theory and Experiment**
- **"Explanations" Reaction Mechanisms**
  - ⋆ **Colinear-limit ETQS-Twist Three . . .**
  - ⋆ **ISI/FSI-Twist Two**
- **Color Gauge Invariance in "T-odd" TMDs Distribution & Fragmentation**
- **Process Dependence and Gluonic Poles in TMDs and FFs**
  - ⋆ **Gluonic Poles and TMDs and Fragmentation Functions**
  - ⋆ **Universality and Fragmentation**
- **Sivers, Collins, Boer-Mulders in SIDIS & Drell Yan & e^+e^-**

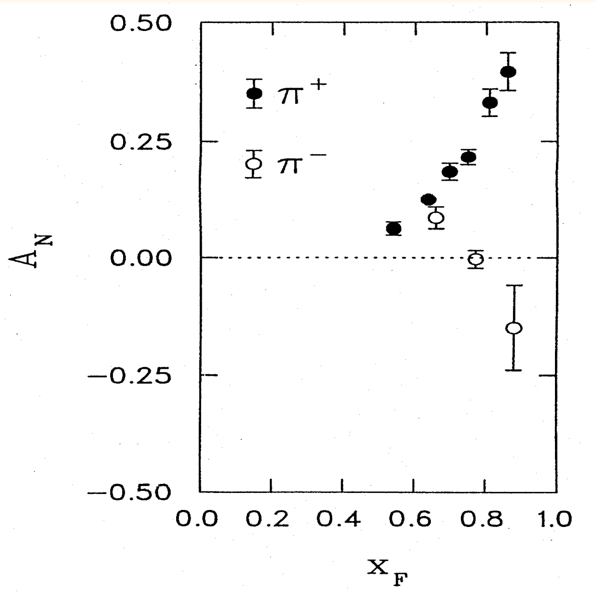
Transverse SPIN Observables SSA (TSSA) $p^{\uparrow}\ p \longrightarrow \pi X$





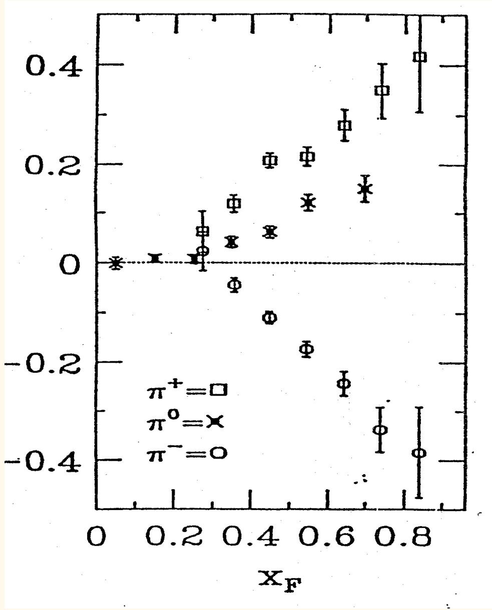
- **Single Spin Asymmetry** $A_N = \frac{\sigma^{\uparrow}(x_F, \boldsymbol{p}_\perp) - \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)}{\sigma^{\uparrow}(x_F, \boldsymbol{p}_\perp) + \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)} \equiv \Delta\sigma$
- **Rotational invariance** $\sigma^{\downarrow}(x_F, \boldsymbol{p}_\perp) = \sigma^{\uparrow}(x_F, -\boldsymbol{p}_\perp)$
  $\Rightarrow$ ***Left-Right Asymmetry***

$\star$ **Parity Conserving interactions: SSAs "Transverse" Scattering plane**
$\Longrightarrow \Delta\sigma \sim i\mathbf{S}_T \cdot (\mathbf{P} \times \boldsymbol{P}_T^{\pi})$



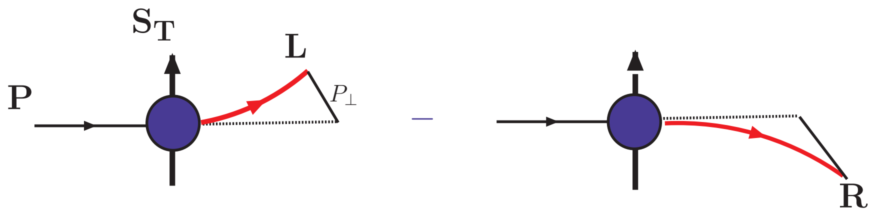
Argonne ZGS, p_{beam} = 12 GeV/c

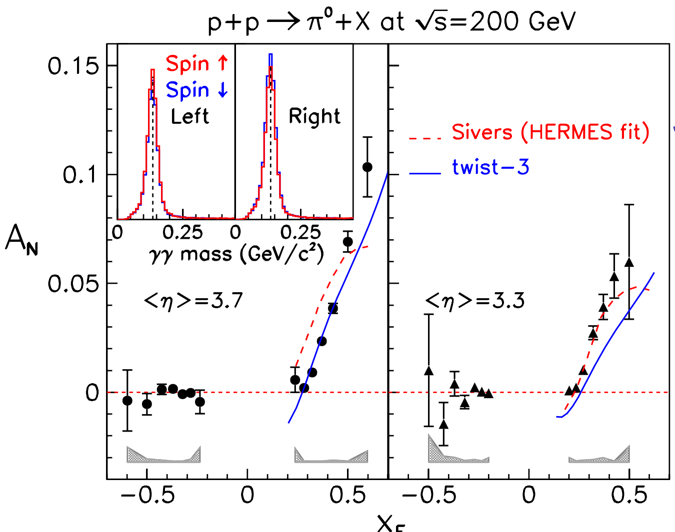
W.H. Dragoset et al.,
PRL 36, 929 (1976)



FNLAL
PLB261, 201 (1991)
PLB264, 462 (1991)
$\sqrt{s}$ =200 GeV

Transverse SSA's at $\sqrt{s}$ = 62.4 GeV at RHIC





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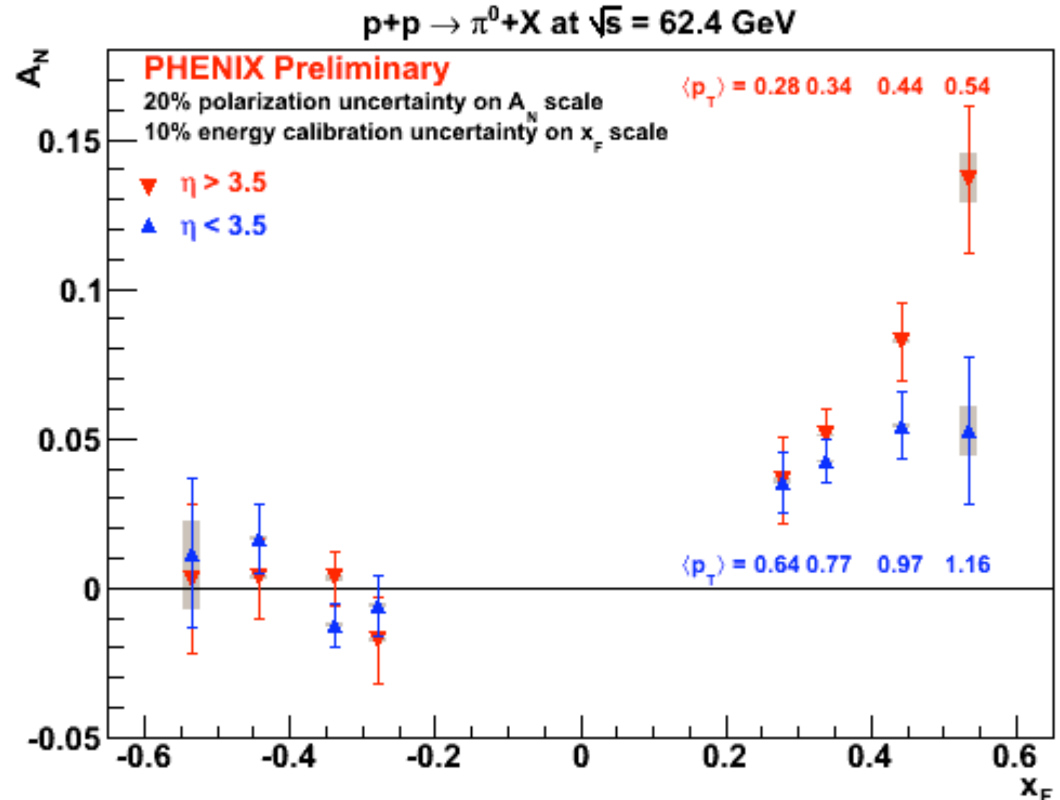
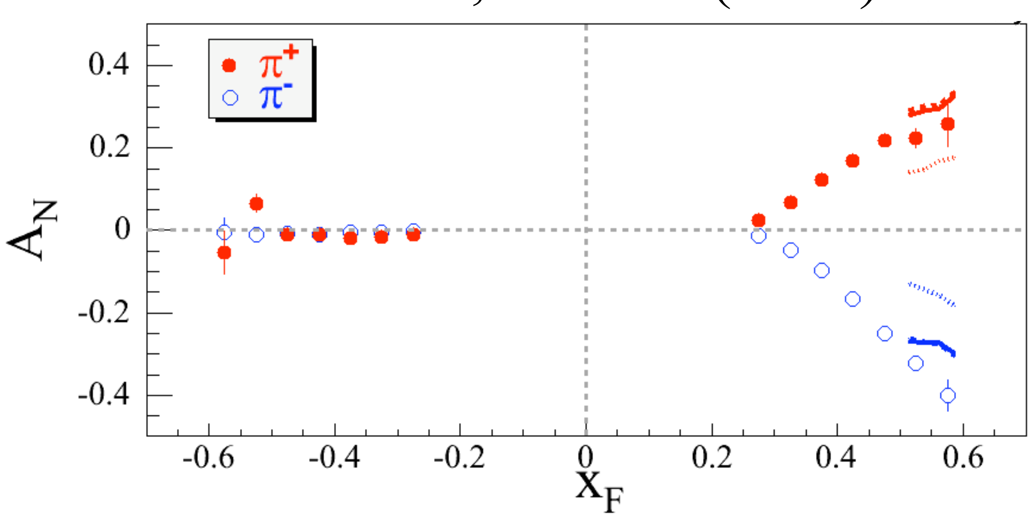


BRAHMS

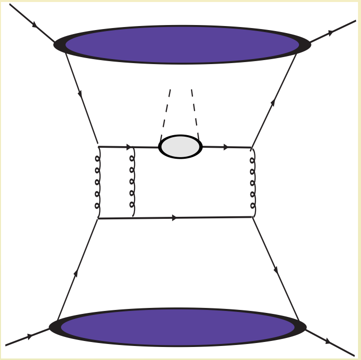
PRL101, 042001 (2008)



Reaction Mechanisms: Co-linear QCD

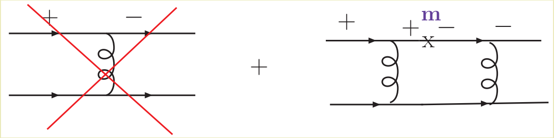
- ⋆ **TSSA requires relative phase btwn *different* helicity amps**

- $|\uparrow / \downarrow\rangle = (|+\rangle \pm i|-\rangle) \Rightarrow \hat{A}_N = \frac{\hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}}{\hat{\sigma}^{\uparrow} + \hat{\sigma}^{\downarrow}} \sim \frac{2\, Im\, f^{*+} f^{-}}{|f^{+}|^2 + |f^{-}|^2}$



- ⋆ **Co-linear factorized QCD-parton dynamics**
  $\Delta\sigma^{pp^{\uparrow}\to\pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q\to\pi}$
  Requires helicity flip-hard part $\Delta\hat{\sigma} \equiv \hat{\sigma}^{\uparrow} - \hat{\sigma}^{\downarrow}$

- **QCD interactions conserve helicity**
  $m_q \to 0$ **and Born amplitudes real**

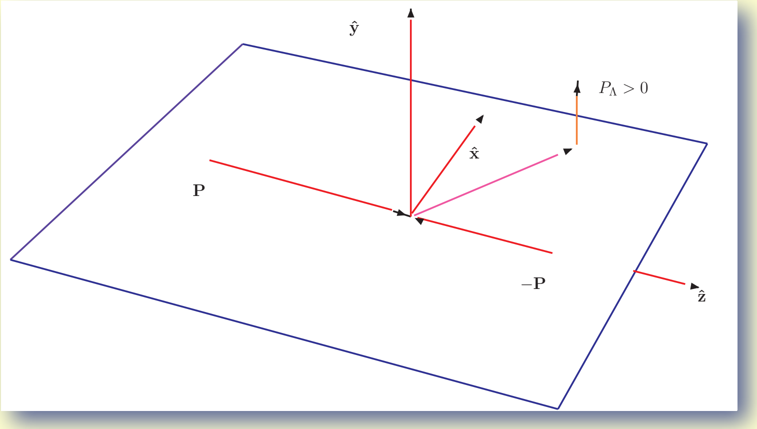


- ⋆ $A_N \sim \frac{m_q \alpha_s}{E}$ **Kane, Repko, PRL:1978**

- ⋆ **M. Anselmino hep-ph/0201150**
  "This makes single spin asymmetries in the partonic interactions entirely negligible"

QCD test-Λ Production $pp \longrightarrow \Lambda^{\uparrow}\ X$

$$P_\Lambda = \frac{\sigma^{pp\to\Lambda^\uparrow X} - \sigma^{pp\to\Lambda^\downarrow X}}{\sigma^{pp\to\Lambda^\uparrow X} + \sigma^{pp\to\Lambda^\downarrow X}}$$



- Need strange quark to polarize a Λ

Interference of loops and tree level Phases in *hard part* $\Delta\hat{\sigma}$

Dharmartna & Goldstein PRD 1990

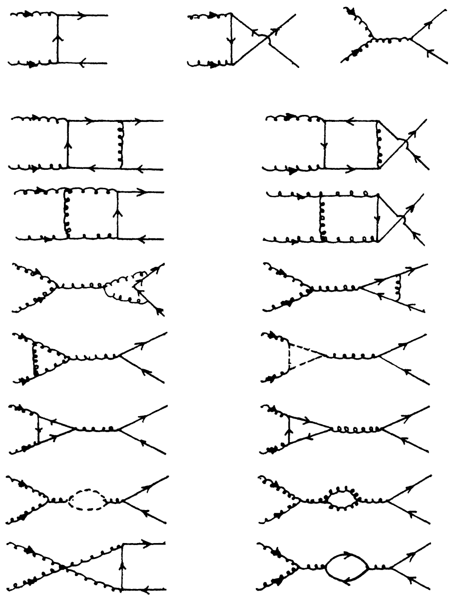


FIG. 1. Feynman diagrams for gluon fusion, $g+g\to s+\bar{s}$. In the second order, only the diagrams which contribute to the imaginary amplitude are shown.

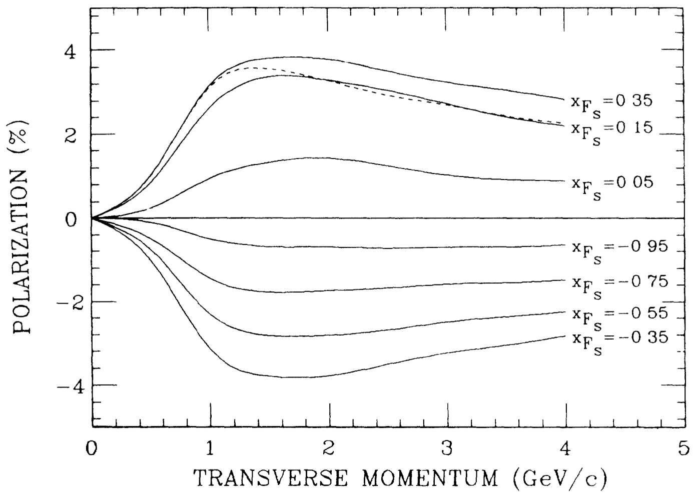


FIG. 4. Strange-quark polarization in the proton c.m. frame, $P_{\text{c.m.}}=14$ GeV/c(400-GeV beam), after the convolution for the initial state gluons. x_{F_s} is the Feynman x for the strange quark. Dashed curve corresponds to $P_{\text{c.m.}}=30.6$ GeV/c.

Early Experiment-Λ Production $pp \to \Lambda^{\uparrow} X$

$$P_\Lambda = \frac{\sigma^{pp\to\Lambda^{\uparrow}X} - \sigma^{pp\to\Lambda^{\downarrow}X}}{\sigma^{pp\to\Lambda^{\uparrow}X} + \sigma^{pp\to\Lambda^{\downarrow}X}}$$

Bunce. . . Heller PRL:1976. . . 1983

- **Experiment *at odd with this result***

P_Λ **in** $p\,p$ **and** $p\,Be$ **scattering-Fermi Lab**

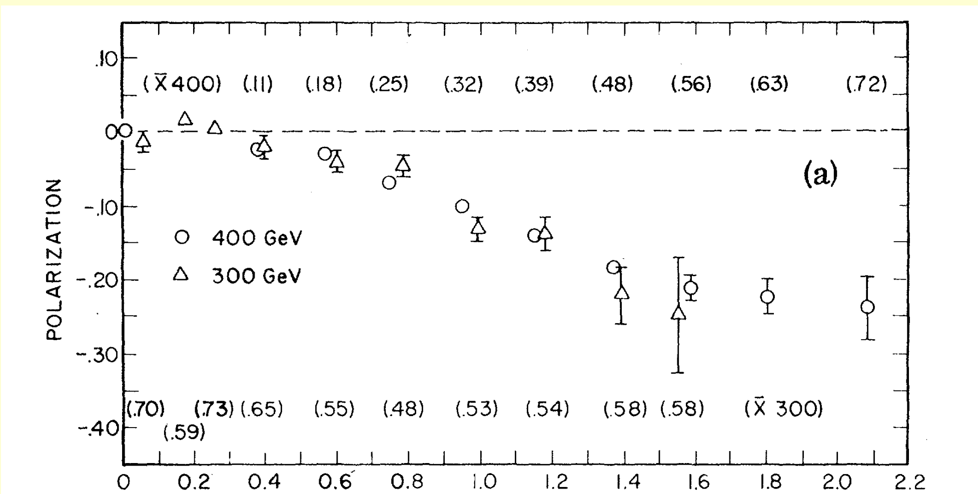
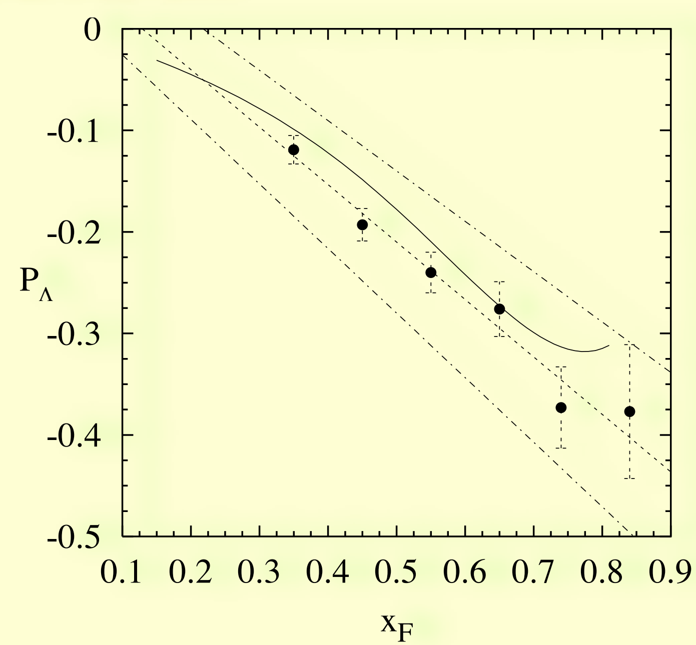


FIG. 3. (a) Λ^0 polarization from this experiment compared to that from 300-GeV incident protons from Ref. 1 as a function of p_T. The number in parentheses is the average value of x for that point. (b) Λ^0 and $\overline{\Lambda}^0$ polarization from this experiment. The polarization is defined as positive along $\hat{n} = (\vec{k}_p \times \vec{k}_\Lambda)/|\vec{k}_p \times \vec{k}_\Lambda|$.

- **Non-perturbative origin many theorists. . .**

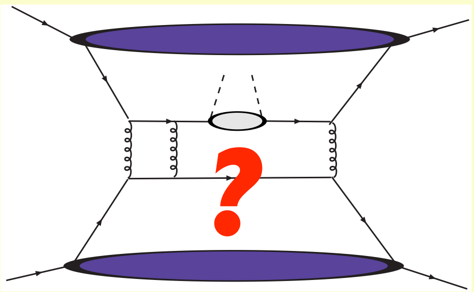
M. Anselmino, D. Boer, U. D'Alesio, and F. Murgia, Phys. Rev. D **63**, 054029 (2001).

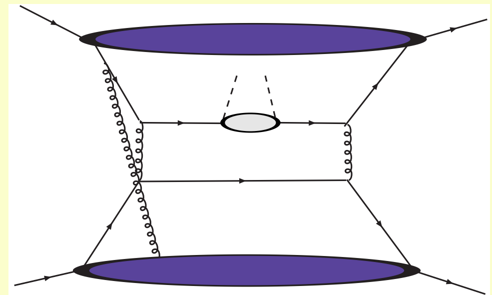


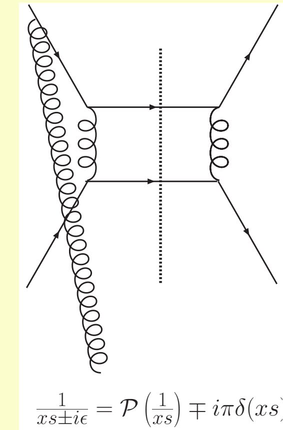
Daniel's talk tomorrow....

$Q \sim P_T >> \Lambda_{qcd}$ Co-linear Twist Three Mechanism

Phases in *soft* poles of propagator in hard subprocess Efremov & Teryaev :PLB 1982





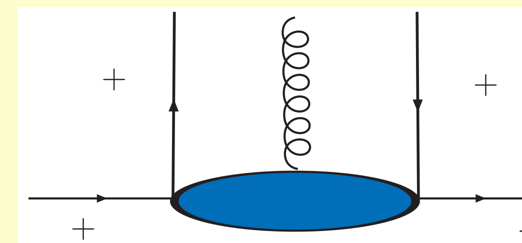


$\star$ **Get helicity flips and phases** $m_q \to \sim M_H$

- $\Delta\sigma \sim f_a \otimes T_F \otimes H_{ETQS} \otimes D^{q\to\pi}$

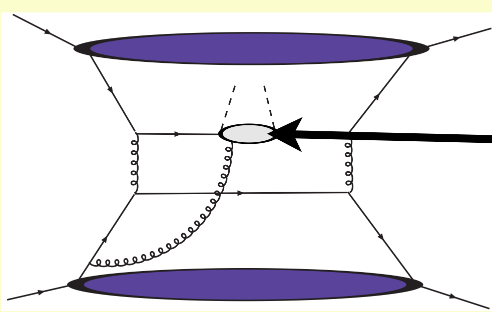
Qiu-Sterman:PLB 1991, 1999, Koike et al. PLB 2000. . . 2007,

Ji,Qiu,Vogelsang,Yuan:PR 2006,2007. . .



quark-gluon-quark correlator

Transversity in pp Koike 2002



. . .

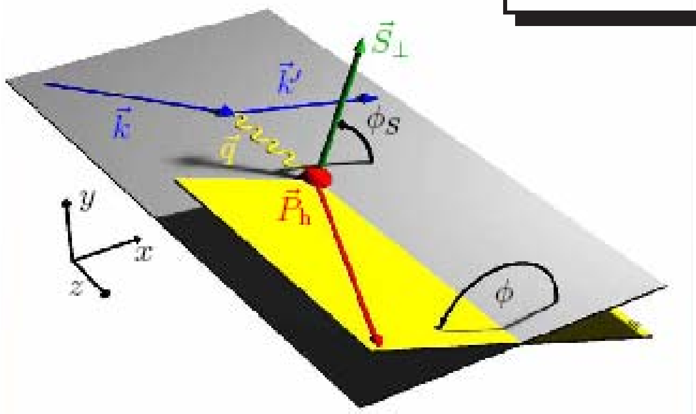
$$\Delta\sigma \sim \delta q(x) \otimes f(x') \otimes \hat{E}(z_1, z_2) \otimes \hat{\sigma}$$

quark-gluon-quark correlator-frag

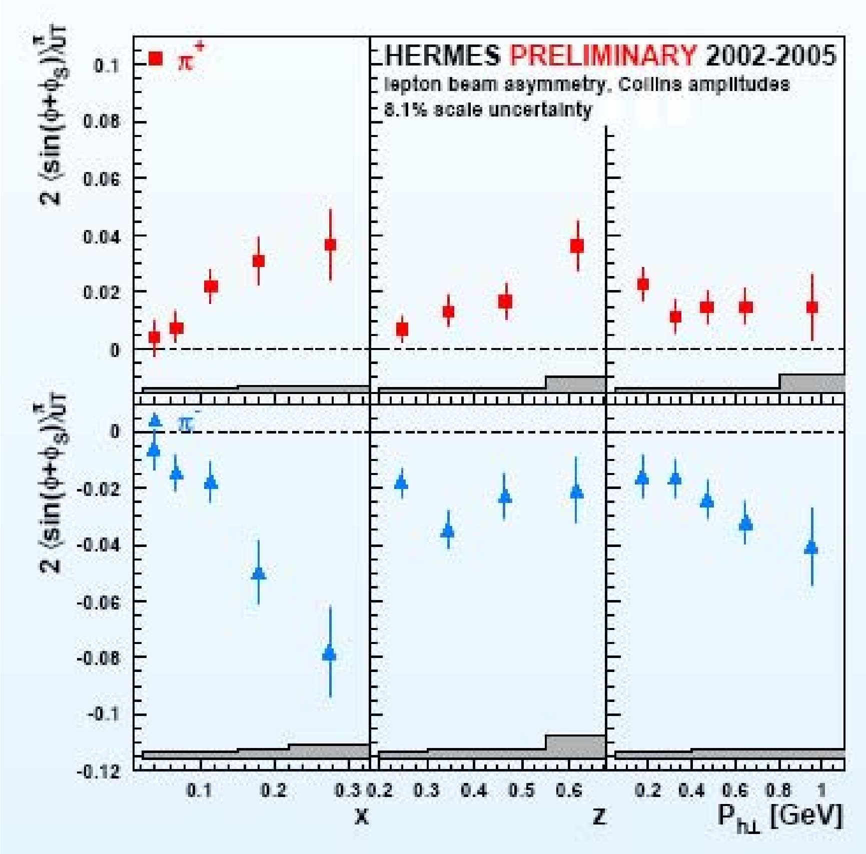
$$\Delta\sigma \sim \delta q(x) f(x') \otimes \hat{H}(z_1, z_2) \otimes \hat{\sigma}$$

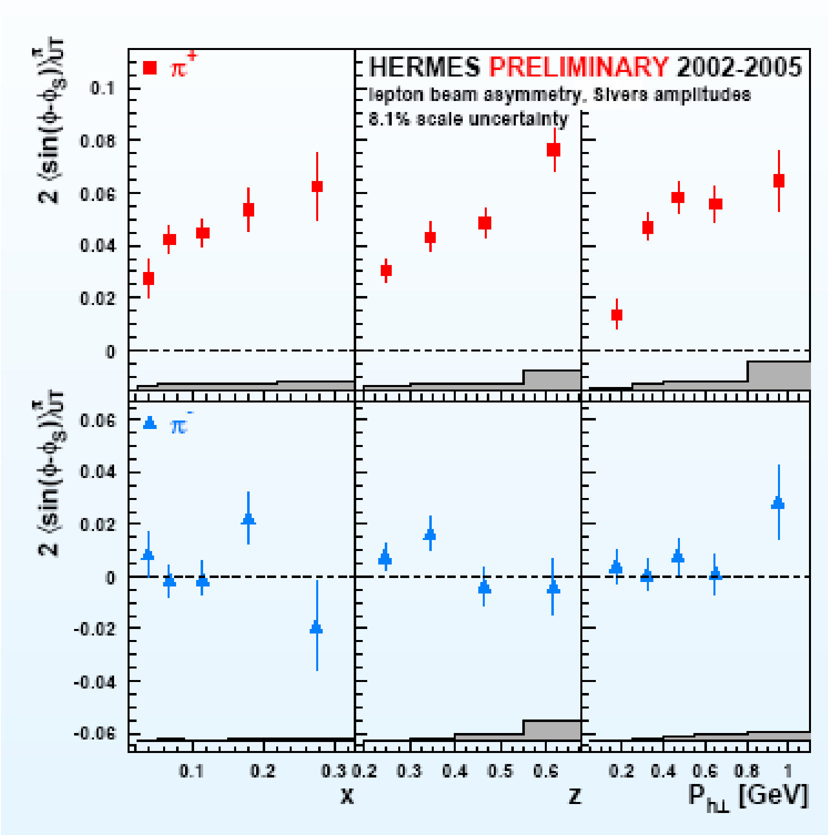
related to Yuan, Zhou hep/ph-0903.4680

Sensitivity of $p_T \sim k_\perp << \sqrt{Q^2}$



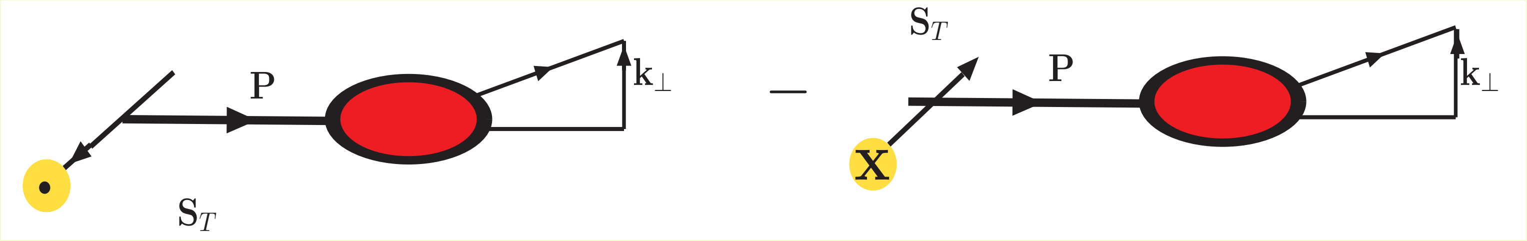
HERMES $ep^\uparrow \Rightarrow \pi\, X$





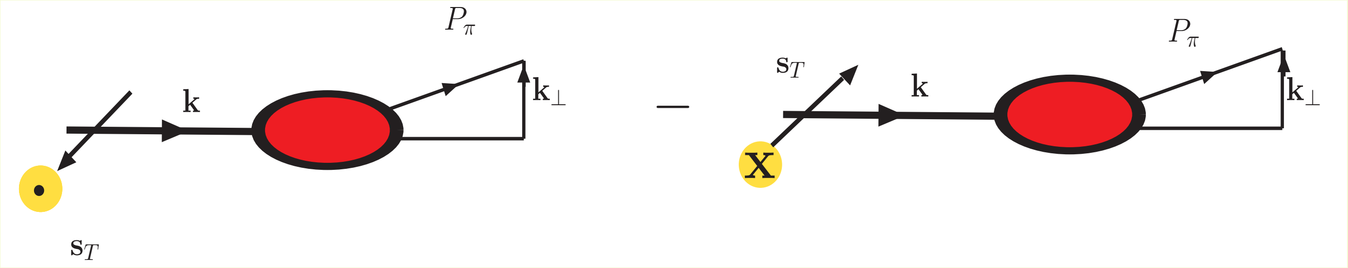
Sensitivity to $p_T \sim k_\perp << \sqrt{Q^2}$ TSSAs thru "T-Odd" TMD

- **Sivers PRD: 1990** TSSA is associated w/ correlation *transverse* spin and momenta in initial state hadron



$$\Delta\sigma^{pp^\uparrow \to \pi X} \sim D \otimes f \otimes \Delta f^\perp \otimes \hat{\sigma}_{Born} \quad \Rightarrow \quad i\boldsymbol{S}_T \cdot (\boldsymbol{P} \times \boldsymbol{k}_\perp) f_{1T}^\perp(x, \boldsymbol{k}_\perp)$$

- **Collins NPB: 1993** TSSA is associated with *transverse* spin of fragmenting quark and transverse momentum of final state hadron



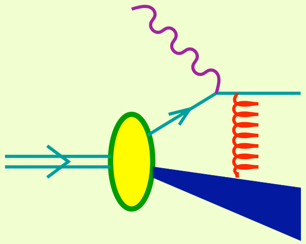
$$\Delta\sigma^{ep^\uparrow \to e\pi X} \sim \Delta D^\perp \otimes f \otimes \hat{\sigma}_{Born} \quad \Rightarrow \quad i\boldsymbol{s}_T \cdot (\boldsymbol{P} \times \boldsymbol{p}_\perp) H_1^\perp(x, \boldsymbol{p}_\perp)$$

Mechanism FSI produce phase in TSSAs-*Leading Twist*

Brodsky, Hwang, Schmidt PLB: 2002

SIDIS w/ transverse polarized nucleon target

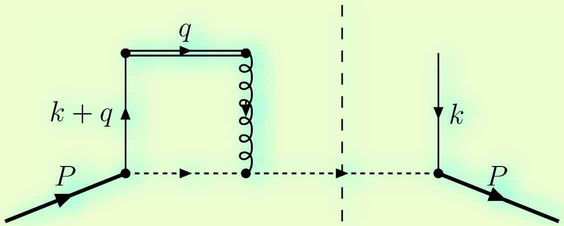
$$e\ p^{\uparrow} \rightarrow e\pi X$$



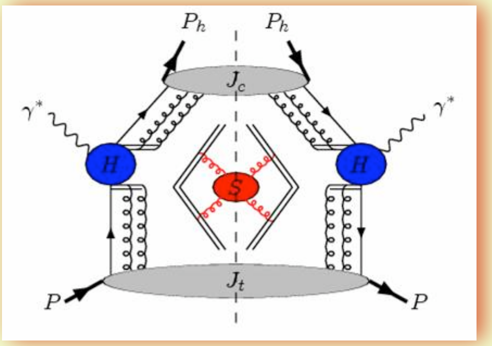
Ji, Yuan PLB: 2002 -Sivers fnct. FSI emerge from Color Gauge-links

Collins PLB 2002

L.G & Goldstein 2002, 2003
Boer-Mulders Fnct,



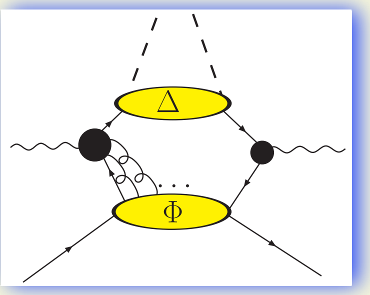
Ji, Ma, Yuan: PLB, PRD 2004, 2005 Extend factorization of CS-NPB: 81



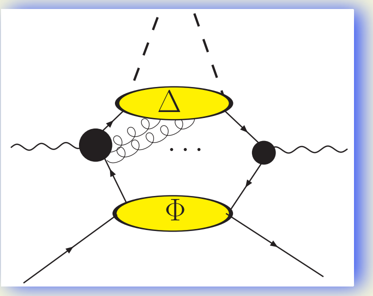
T-Odd Effects From Color Gauge Inv. Factorized QCD-Wilson Line

Boer, Mulders: NPB 2000, & Pijlman (BPM) NPB 2003, Belitsky Ji Yuan NPB 2003

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-, \xi_T)|P\rangle|_{\xi^+=0}$$



$$\Delta^{[\mathcal{U}[\mathcal{C}]]}(z, k_T) = \int \frac{d\xi^+ d^2\xi_T}{4z(2\pi)^3} e^{ik\cdot\xi} \langle 0\, |\mathcal{U}^{[C]}_{[0,\xi]}\psi(0)|x; P_h\rangle\langle x; P_h|\overline{\psi}(\xi^+, \xi_T)|0\rangle|_{\xi^-=0}$$

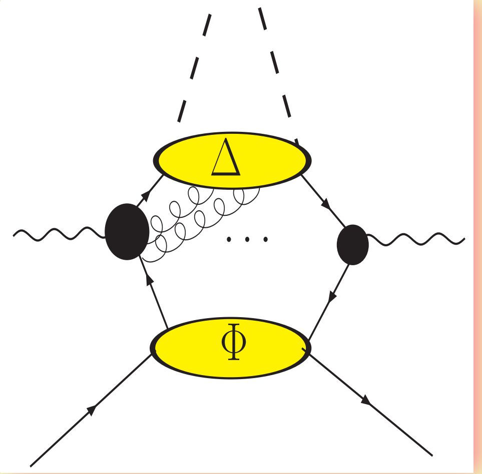


T-Odd Effects From Color Gauge Inv. via Wilson Line

Amsterdam group Boer Mulders, Pijlman, Bomhof et al. 2003 - 2008 ....

- *Sub-class of interactions* of *colinear & transverse* gluons re-summed to render *physical process* color gauge invariant

- Gauge link emerges from resummation of gluon ISI and FSI *btw. active quark and hadron remnants*



$$\rightarrow U^{[C]}_{[\eta,\xi]} = \mathcal{P}exp(-ig \int_C ds^\mu A_\mu)$$

etc . . .

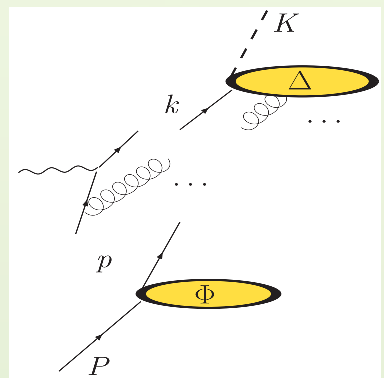
- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

Diagramatic "Factorization" TMD Correlators Gauge Links

Politzer NPB 80, Ellis et al. NPB 82, Mulders et al. 1997

- Restricts hadrons well sep. in momentum phase-space $P \cdot K \sim p \cdot k \sim Q^2$
- Inside correlator momenta are soft $P \cdot p \sim P^2 = M^2$
- Partons involved decomposed according to "Sudakov" P and n vectors

$$p = \underbrace{x P^\mu}_{\sim Q} + \underbrace{p_T^\mu}_{\sim M} + \underbrace{\sigma n^\mu}_{\sim M^2/Q}$$



$$\begin{aligned} k &= \frac{1}{z} K^\mu + k_T^\mu + \sigma_h n_h^\mu \\ p &= x P^\mu + p_T^\mu + \sigma n^\mu \\ & \quad\ \sim Q \quad \sim M \quad \sim M^2/Q \end{aligned}$$

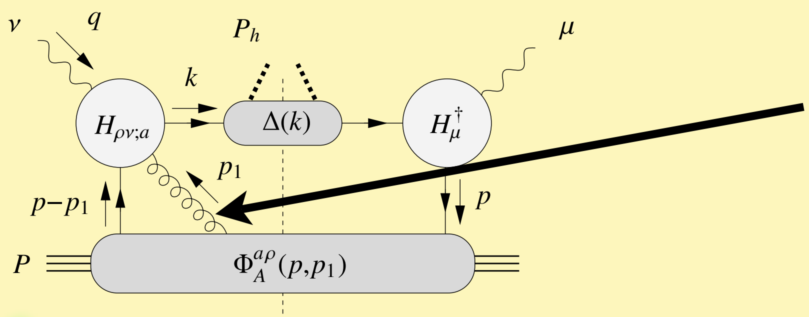
$$n^2 = 0, \quad P \cdot n = 1, \quad K \cdot n_h = 1, \quad \sigma = (p \cdot P - x M^2) \sim M^2, \quad \sigma_h \sim M_h^2 \quad \ldots$$

TMD-Integrate over $P \cdot p$ $\qquad \Phi^{[\mathcal{U}[\mathcal{C}]]}(x, p_T) = \int d(p \cdot P) \Phi(p, P)$

- Organizes Twist Expansion
- Determines gauge-link by summing collinear & transverse gluon interactions btwn. soft & hard

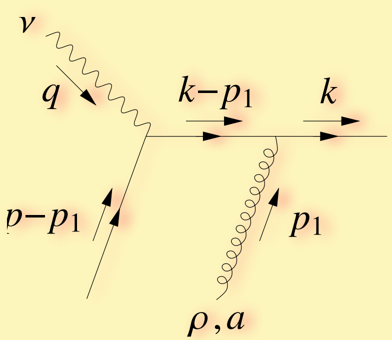
Summation of gluons from soft to hard at leading twist involves gluons collinear to hadron's momentum

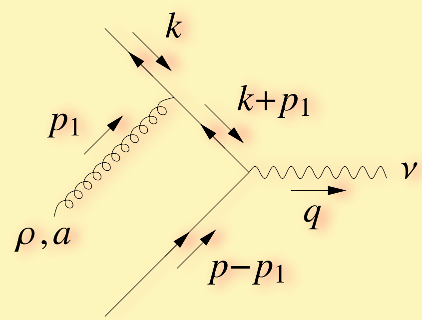
- $A^\mu \propto (A \cdot n) P^\mu + A_T^\mu$



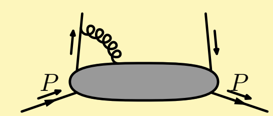
Bomhof, Mulders, Pijlman, Boer et al.
Gauge link determined by resumming all gluon interactions btwn soft and hard

$$\frac{1}{2M}\int d^4p\, d^4k\, d^4p_1\; \delta^4(p+q-k)\,\mathrm{Tr}\left[\,\Phi_A^{a\rho}(p,p_1)\,H_\mu^\dagger(p,k)\,\Delta(k)\,H_{\rho\nu;a}(p,k;p_1)\,\right]$$





Note(!) interactions between lines which are connected to "same" jet absorbed into matrix element, e.g. Pijlman Ph.D. thesis 2006



The hard tree amplitudes in SIDIS and DY dressed with leading co-linear gluon insertions "eikonalize". Convoluting this hard amplitude with soft factors determines "[C]" factors

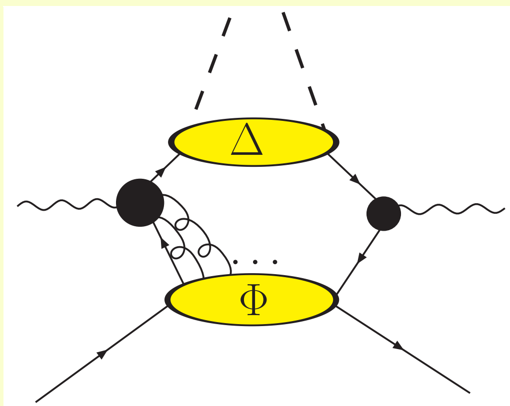
$$\longrightarrow \quad \int d^4p\, d^4k\; \delta^4(p+q-k)\;\mathrm{Tr}\left[\,\Phi_{(g)}^{[U^n_{[\infty;\xi]}]}(p)\,H_\mu^\dagger(p,k)\,\Delta(k)\,H_\nu(p,k)\,\right]$$

- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

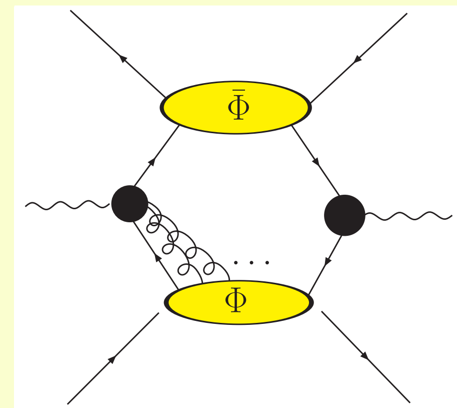
T-Odd Effects From Color Gauge Inv. via Wilson Line

Prediction of QCD $\quad f^{\perp}_{1T_{SIDIS}}(x,k_T) = -f^{\perp}_{1T_{DY}}(x,k_T)$

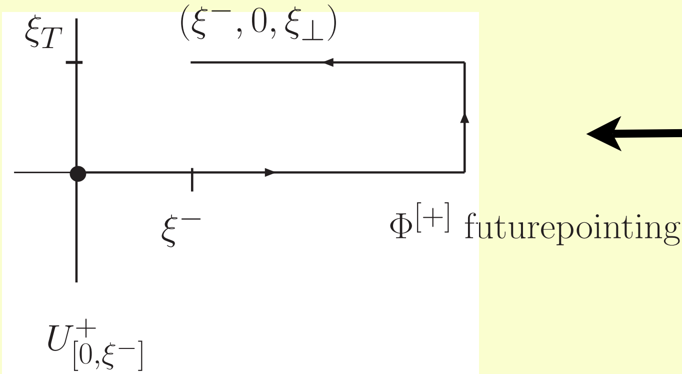
Process Dependence Collins PLB 02, Brodsky, Hwang, Schmidt NPB 02



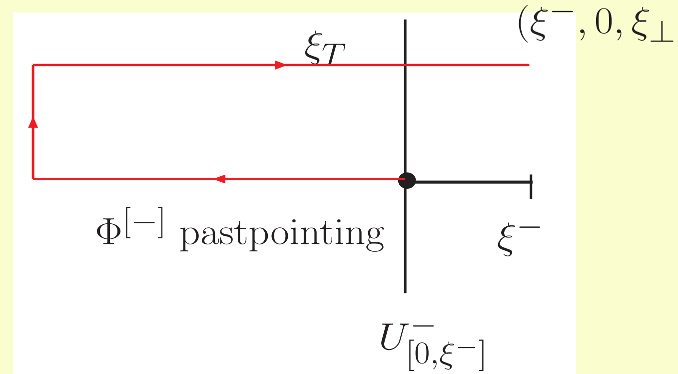
$d\sigma = L_{\mu\nu}\mathcal{W}^{\mu\nu} \Rightarrow$



SIDIS Hadronic Tensor **Drell-Yan Hadronic Tensor**

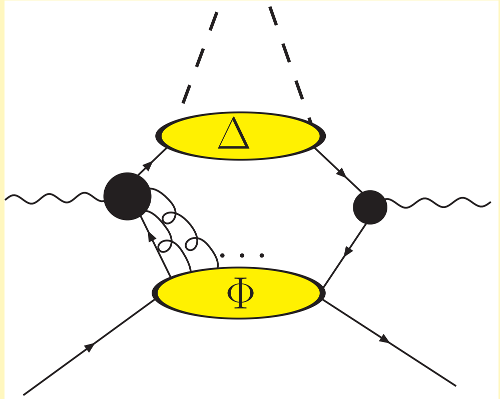


$\xleftrightarrow{\ \boldsymbol{T}\ }$



$$\Phi^{[+]*}(x,p_T) = i\gamma^1\gamma^3\Phi^{[-]}(x,p_T)i\gamma^1\gamma^3$$

TMD Correlator in SIDIS expanded in DFs and FFs which are T-odd and even



$$\Delta(z,\boldsymbol{k}_\perp)=\frac{1}{4}\{D_1(z,\boldsymbol{k}_\perp)\not{n}_- + H_1^\perp(z,\boldsymbol{k}_\perp)\frac{\sigma^{\alpha\beta}k_{\perp\alpha}n_{-\beta}}{M_h} + D_{1T}^\perp(z,\boldsymbol{k}_\perp)\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_-^\nu k_\perp^\rho S_{hT}^\sigma}{M_h} + \cdots\}$$

$$\Phi(x,\boldsymbol{p}_\perp)=\frac{1}{2}\{f_1(x,\boldsymbol{p}_\perp)\not{n}_+ + h_1^\perp(x,\boldsymbol{p}_\perp)\frac{\sigma^{\alpha\beta}p_{T\alpha}n_{+\beta}}{M} + f_{1T}^\perp(x,\boldsymbol{p}_\perp)\frac{\epsilon^{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_\perp^\rho S_T^\sigma}{M}\cdots\}$$

SIDIS cross section

$$
\begin{aligned}
d\sigma^{\ell N\to\ell\pi X}_{\{\lambda,\Lambda\}} \quad \propto \quad & f_1 \otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1 + \frac{k_\perp}{Q} f_1 \otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1\cdot\cos\phi \\
+ \quad & \left[\frac{k_\perp^2}{Q^2} f_1 \otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1 + h_1^\perp \otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^\perp\right]\cdot\cos 2\phi \\
+ \quad & |S_T|\cdot h_1 \otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^\perp\cdot\sin(\phi+\phi_S) \quad \text{Collins} \\
+ \quad & |S_T|\cdot f_{1T}^\perp \otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes D_1\cdot\sin(\phi-\phi_S) \quad \text{Sivers} \\
+ \quad & |S_L|\cdot h_{1L}^\perp \otimes d\hat{\sigma}^{\ell q\to\ell q}\otimes H_1^\perp\cdot\sin(2\phi) \quad \text{Kotzinian}-\text{MuldersPLB}
\end{aligned}
$$

Boer-Mulders

transversity

Azimuthal asymm. corresponds to transv. moments of correlator In factorized processes

$$\Phi_\partial^{\alpha[\mathcal{U}]}(x) = \int d^2k_T k_T^\alpha \Phi^{[\mathcal{U}]}(x, k_T).$$

Decomposes $\Phi_\partial^{\alpha\,[\mathcal{U}]}(x) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[\mathcal{U}]}\,\pi\Phi_G^\alpha(x,x),$

T-even

T-odd

$$\epsilon_T^{ij} k_\perp^i S_T^j f_{1T}^{\perp(1)}(x, k_\perp^2) \sim \int d^2k_T\, k_T^i\, \frac{1}{2}\left[\mathrm{Tr}[\gamma^+\Phi(\vec{S}_T)] - \mathrm{Tr}[\gamma^+\Phi](-\vec{S}_T)\right]$$

$$A_{UT}^{P_{h\perp}/M}\sin(\phi - \phi_S)(x,z) = \frac{(-2)\sum_a e_a^2 x f_{1T}^{\perp(1)a}(x) D_1^a(z)}{\sum_a e_a^2 x f_1^a(x) D_1^a(z)}$$

$$2\pi\Phi_G^\alpha(x,x) = (ih_1^{\perp(1)}\frac{1}{2}[\not{P},\gamma^\alpha] + \epsilon_T^{\alpha\,S_T} f_{1T}^{\perp(1)}(x)\not{P})$$

Weighted Cross Sections contain ETQS Functions LINK BTW TWO Pictures!

In azimuthal asymm. one uses transv. moments of the correlator

$$\Phi_\partial^{\alpha[\mathcal{U}]}(x) = \int d^2k_T\, k_T^\alpha \Phi^{[\mathcal{U}]}(x, k_T).$$

Decomposes $$\Phi_\partial^{\alpha\,[\mathcal{U}]}(x) = \tilde{\Phi}_\partial^\alpha(x) + C_G^{[\mathcal{U}]}\,\pi\Phi_G^\alpha(x,x),$$

T-even

T-odd

- **For the weighted cross sections the process dependence is in** *gluonic pole factors* *Bomhof, Pijlman, Mulders 2004-2008 JHEP,NPB...*

$$\begin{aligned}
< q_T^\alpha d\sigma > \quad &\sim \quad \tilde{\Phi}_\partial^{\alpha[\mathcal{U}]}(x)\hat{\sigma}_{lq\to lq}\Delta(z) + C_G^{[U(C)]}\pi\Phi_G^{\alpha[\mathcal{U}]}(x,x)\hat{\sigma}_{lq\to lq}\Delta(z) \\
< q_T\ \sigma_{lH\to lhX}^{\text{Sivers}} > \quad &\sim \quad +f_{1T}^{\perp(1)}(x)\hat{\sigma}_{lq\to lq}D_1(z) \\
< q_T\ \sigma_{H\bar{H}\to l\bar{l}X}^{\text{Sivers}} > \quad &\sim \quad -f_{1T}^{\perp(1)}(x_1)\hat{\sigma}_{q\bar{q}\to l\bar{l}}\bar{f}_1(x_2) \\
f_{1T}^{\perp(1)}(x) \quad &= \quad -\frac{g}{2M}T_F(x)
\end{aligned}$$

Boer,Pijlman,Mulders 2003 NPB

Similarly unintegrated fragmentation there are in principle "two" types of gauge links

However more subtle!!! -Two types of T-odd effects

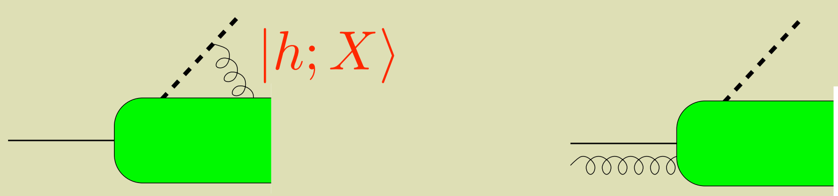
Reliability of Transversity Extraction Universality of Collins Fragmentation Function

$$\Delta_{\partial}^{\alpha\,[\mathcal{U}]}(z) \quad = \quad \int d^2k_T\; k_T^{\alpha}\Delta^{[\mathcal{U}]}(z,k_T) = \tilde{\Delta}_{\partial}^{\alpha}\left(\tfrac{1}{z}\right) + C_G^{[\mathcal{U}]}\,\pi\Delta_G^{\alpha}\left(\tfrac{1}{z},\tfrac{1}{z}\right)$$

$$\Delta^{[-]}(z,k_T) = \int \frac{d\xi^+ d^2\xi_T}{4z(2\pi)^3}\; e^{ik\cdot\xi}\,\langle 0\;|U^{[-]}_{[-\infty,0]}\psi(0)|x;P_h\rangle\langle x;P_h|\overline{\psi}(\xi^+,\xi_T)U^{[-]}_{[\xi,-\infty]}|0\rangle|_{\xi^-=0}$$

But no such constraint under time reversal

$$\Delta^{[+]*}(x,p_T) \neq i\gamma^1\gamma^3\Delta^{[-]}(x,p_T)i\gamma^1\gamma^3$$



$$\Delta_{\partial}^{\alpha\,[C]}(z) \quad = \quad \tilde{\Delta}_{\partial}^{\alpha[\not{C}]}\left(\tfrac{1}{z}\right) + C_G^{[U(C)]}\,\pi\Delta_G^{\alpha[\not{C}]}\left(\tfrac{1}{z},\tfrac{1}{z}\right)$$

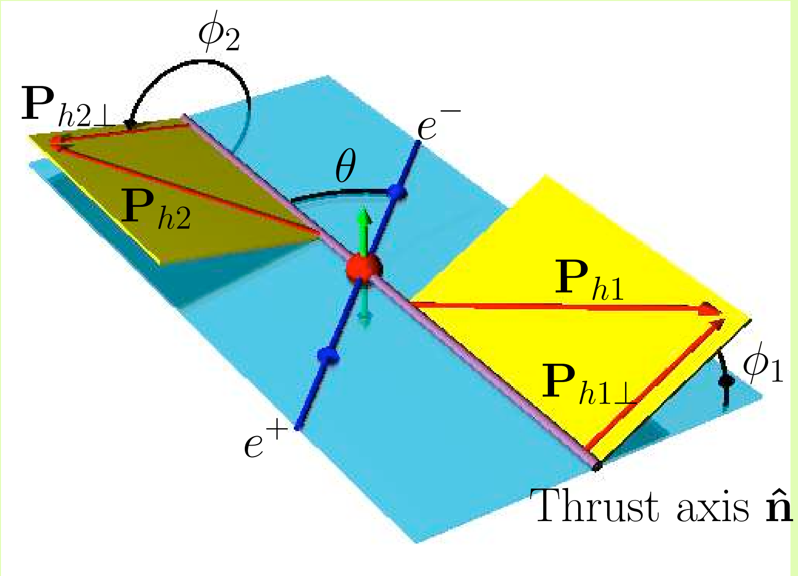
T-odd-FSI **_T-odd-Gauge link_**

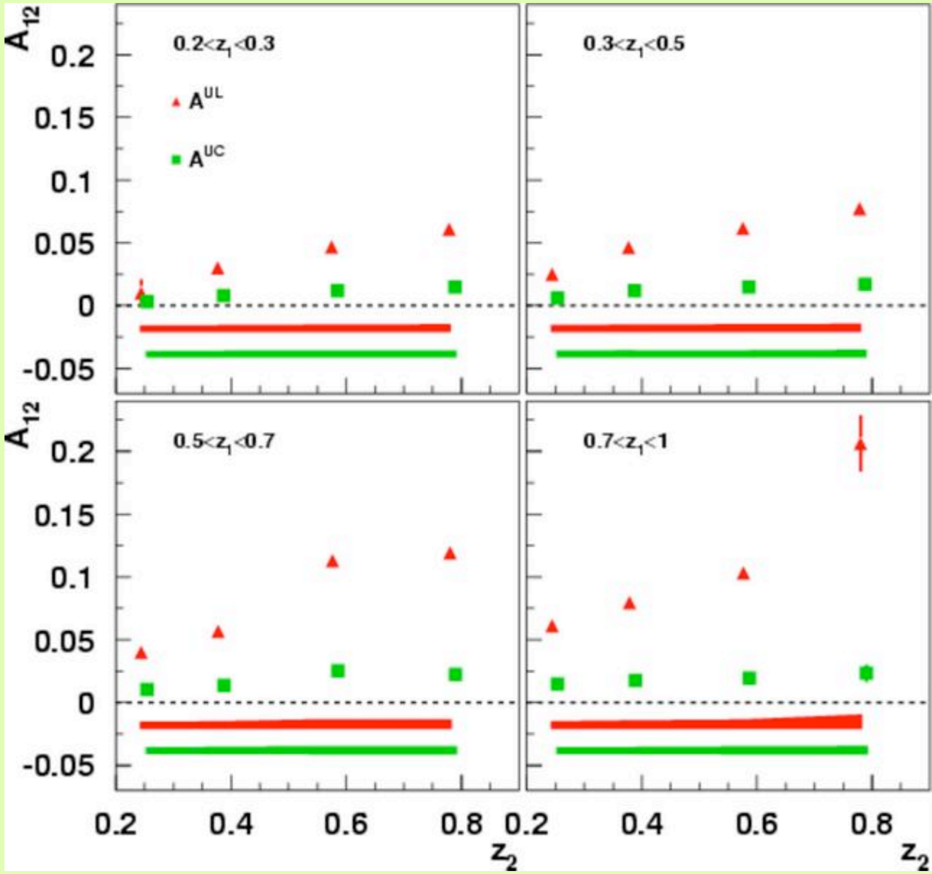
Reliability of Transversity Extraction Universality of Collins Fragmentation Function

Belle KEKB measurement of the Collins Frag. Function PRL 2006 & arXiv:0805.2975

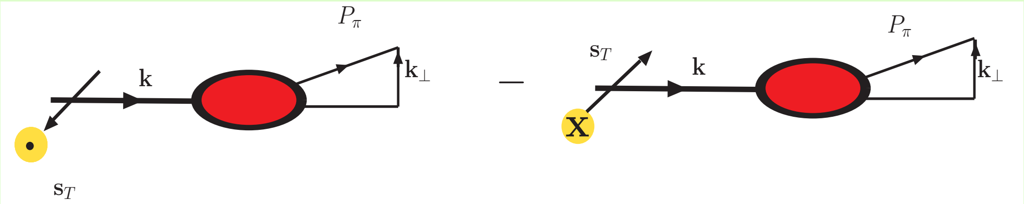
$$\frac{d\sigma\left(e^+e^- \to h_1 h_2 X\right)}{d\Omega dz_1 dz_2 d^2 q_T} = \cdots B(\Theta)\cos(\varphi_1 + \varphi_2) H_1^{\perp[1]}(z_1)\overline{H}_1^{\perp[1]}(z_2)$$

$$B(\Theta) \stackrel{cm}{=} \frac{1}{4}\sin^2\Theta$$





Ralf Seidl EIC Workshop, Hampton, VA May 08



- Collins NPB: 1993 **TSSA is associated with** *transverse* **spin of fragmenting quark and transverse momentum of final state hadron**

$$D_{h/q^\uparrow}\left(z, K_T^2\right) = D_1^q\left(z, K_T^2\right) + H_1^{\perp q}\left(z, K_T^2\right)\frac{(\hat{\boldsymbol{k}} \times \boldsymbol{K}_T)\cdot \boldsymbol{s}_q}{zM_h},$$

$$\frac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^\perp\left(z, k_T^2\right) = \frac{1}{2}\,\mathrm{Tr}\left[\Delta(z, k_T) i\sigma^{i-}\gamma_5\right].$$

$$\Delta_\partial^{\alpha\,[\mathcal{U}]}(z) \quad = \quad \int d^2k_T\; k_T^\alpha \Delta^{[\mathcal{U}]}(z, k_T) = \tilde{\Delta}_\partial^\alpha\left(\tfrac{1}{z}\right) + C_G^{[\mathcal{U}]}\,\pi\Delta_G^\alpha\left(\tfrac{1}{z}, \tfrac{1}{z}\right)$$

Spect. model workbench ISI/FSI in AA & TMDs $h_1^\perp$, $f_{1T}^\perp$, $H_1^\perp$ gluonic poles

- $\not\exists$ *calculation Quark-Quark Correlator in Full QCD*

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$

- Use Spectator Framework Develop a QFT to explore and estimates these effects with gauge links
  - ⋆ BHS FSI/ISI Sivers fnct, -PLB 2002, NPB 2002
  - ⋆ Ji, Yuan PLB 2002 - Sivers Function
  - ⋆ Metz PLB 2002 - Collins Function
  - ⋆ L.G. Goldstein, 2002 ICHEP- Boer Mulders Function
  - ⋆ L.G. Goldstein, Oganessyan TSSA & AAS PRD 2003-SIDIS
  - ⋆ Boer Brodsky Hwang PRD 2003-Drell Yan Boer Mulders
  - ⋆ Bacchetta Jang Schafer 2004- PLB, Flavor-Sivers, Boer Mulders
  - ⋆ Lu Ma Schmidt PLB, PRD, 2004/2005 Pion Boer Mulders
  - ⋆ L.G. Goldstein DY and higher twist, PLB 2007
  - ⋆ LG, Goldstein, Schlegel PRD 2008-Flavor dep. Boer Mulders $\cos 2\phi$ SIDIS
  - ⋆ Conti, Bacchetta, Radici, Ellis, Hwang, Kotzinian 2008 hep-ph . . . !
- Spectator Model “Field Theoretic” used study Universality of T-odd Fragmentation Δ_{ij}
  - ⋆ Metz PLB 2002, Collins Metz PRL 2004
  - ⋆ Bacchetta, Metz, Yang, PBL 2003, Amrath, Bacchetta, Metz 2005,
  - ⋆ Bacchetta, L.G. Goldstein, Mukherjee, PLB 2008
  - ⋆ Collins Qui, Collins PRD 2007,2008
  - ⋆ Yuan 2-loop Collins function PRL 2008
  - ⋆ L.G., Mulders, Mukherjee Gluonic Poles PRD 2008

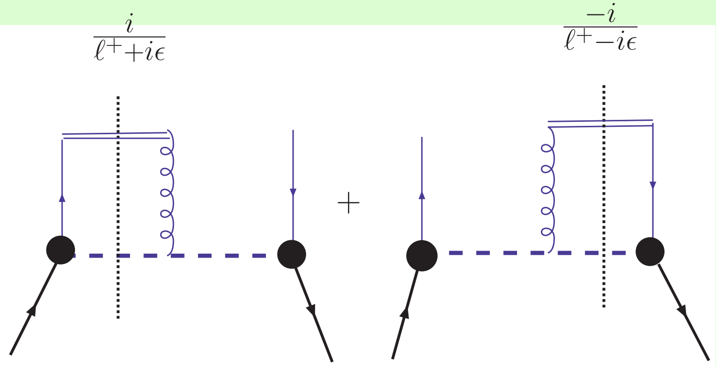
Mechanisms explored thru T-odd Contribution SIDIS and Drell Yan

Impacts pre and "post"-dictions at COMPASS, HERMES, JLAB 6 & 12 GeV FAIR, RHIC, JPARC

$\cos 2\phi$ Asymmetry in SIDIS-"Boer Mulders Effect"

⋆ **Early wk. in spectator framework** Goldstein, L.G., ICHEP 2002; hep-ph/0209085 L.G., Goldstein, Oganessyan, PRD 2003, Boer Brodsky Hwang, PRD 2003

$$h_1^{\perp(s)}(x,k_\perp) = f_{1T}^{\perp(s)}(x,k_\perp)$$



- **Collins, Sivers and Boer Mulders Asymmetries with Gaussian Distribution in $k_\perp$** L.G., Goldstein, Oganessyan, PRD 67 (2003)

$$h_1^\perp(x,k_\perp) \quad = \quad \alpha_s \mathcal{N}_s \frac{M(m+xM)(1-x)}{k_\perp^2 \Lambda(k_\perp^2)} \mathcal{R}(k_\perp^2, x)$$

Revisit Gluonic Poles contributions-Fragmentation & Universality

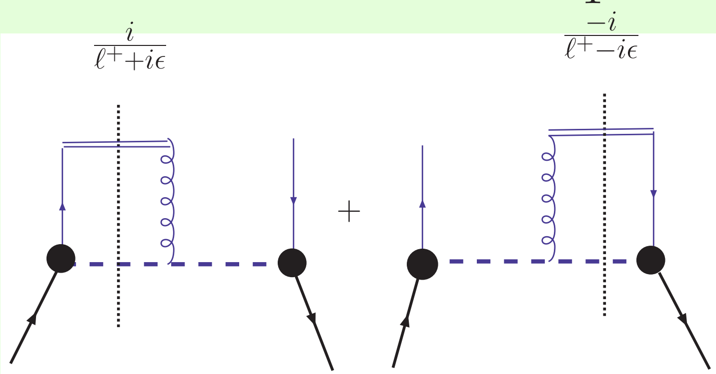
(Gamberg, Mukherjee, Mulders PRD-2008)

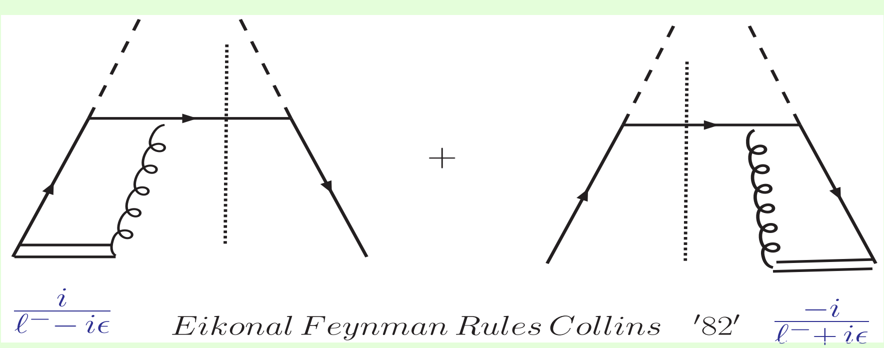
- **By contrast to one and two loop calcs. studying cuts we explore parton correlator with one additional gluon taking the zero $k_1^{\pm} \to 0$ limit; gluonic pole matrix element/Efremov-Terayev-Qiu Sterman Matrix elements**

- **Gluonic Poles Identify the T-odd sources and possible non-universal or process dependent contributions in PDFs and FFs**
  Boer, Pijlman, Mulders NPB 03, Bacchetta, Bomhof, Mulder, Pijlman PRD 05, Bomhof Mulders 07, 08, Bomhof, Mulders, Vogelsang, Yuan PRD 07.

- **In doing so we investigated the "reciprocity" btwn distrb. and frag. functions $x \to 1/z$**

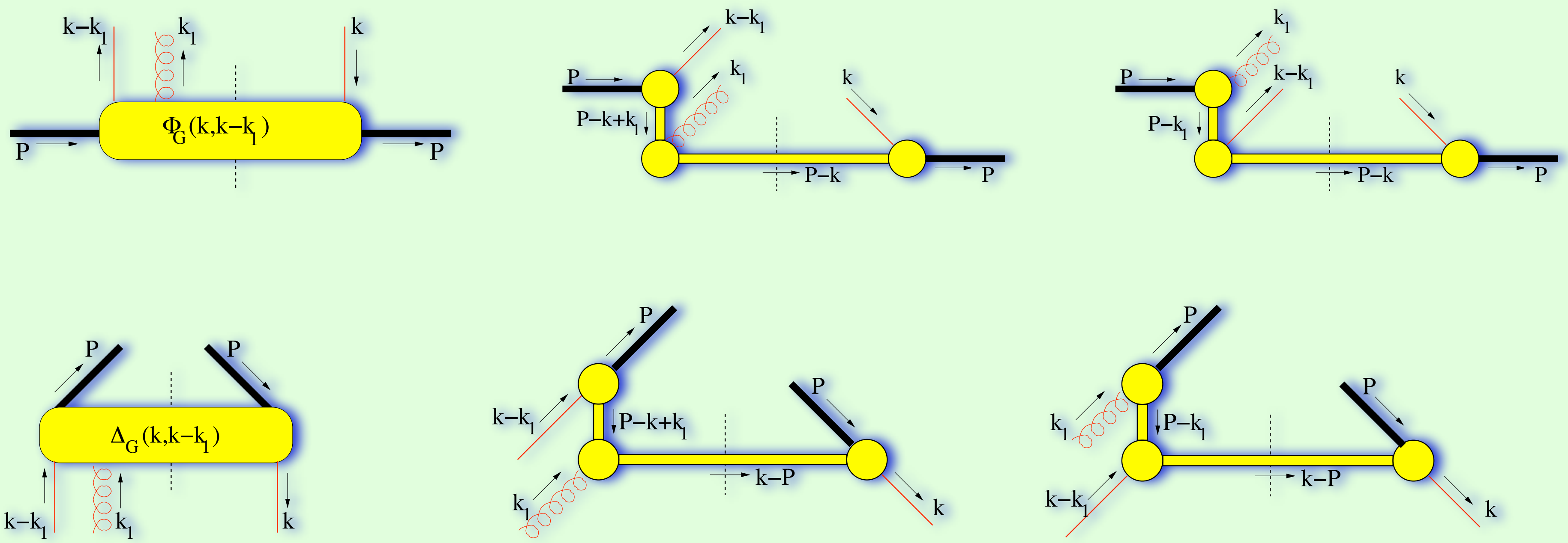
$$h_1^{\perp(1)} \text{ and } H_1^{\perp(1)} \text{ for example}$$



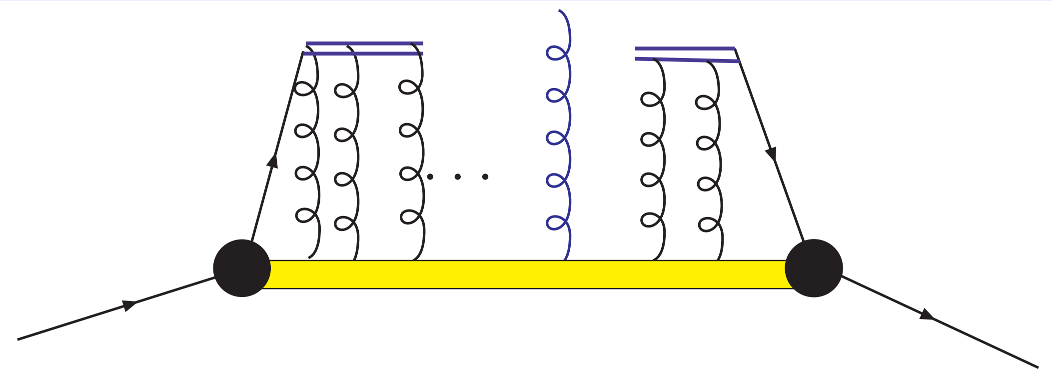


Spectral Analysis Gluonic Poles-Fragmentation

- In this approach rather than integrating over the longitudinal component of the "loop momenta" we look at the limit of a zero gluon momentum in quark-gluon-quark matrix element
- That is considering the multi-parton correlators $\Phi_G(k, k-k_1)$ and $\Delta_G(k, k-k_1)$ in light-cone gauge



$$
\begin{aligned}
\Delta^{\alpha}_{G\,ij}(x, x-x_1) \;=\; & \sum_X \int \frac{d(\xi\cdot P)}{2\pi}\frac{d(\eta\cdot P)}{2\pi}\, e^{i\,x_1(\eta\cdot P)} e^{i\,(x-x_1)(\xi\cdot P)} \\
& \times \langle 0|\mathcal{U}^n_{[0,\eta]}\, gG^{n\alpha}(\eta)\,\mathcal{U}^n_{[\eta,\xi]}\psi_i(\xi)|P,X\rangle \\
& \times \langle P,X|\overline{\psi}_j(0)|0\rangle\Big|_{LC} .
\end{aligned}
$$



Calculate the Weighted Frag. correlator

- The T-odd operator-combination that appears in the transverse moment,

$$\Delta_\partial^\alpha(z;n,C) = \int d^2k_T k_T^\alpha \Delta(x,k_T;n,C) = \tilde{\Delta}_\partial^\alpha(z) + \pi\Delta_G^\alpha(z)$$

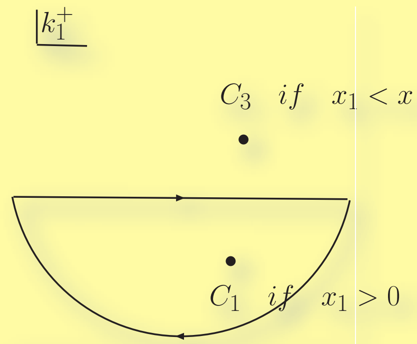
yields the gluonic pole matrix element which is characterized by the difference of the transverse gluon field at $\pm\infty$,

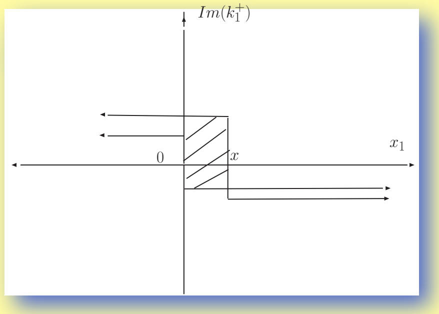
$$\begin{aligned}\Delta_{G\,ij}^\alpha(z;n,C) &= \sum_X \int \frac{d(\xi\cdot P_h)}{2\pi} e^{ik\cdot\xi} \langle 0|\left(A_T^\alpha(\infty) - A_T^\alpha(-\infty)\right) \\ &\quad \times \psi_i(\xi)|P_h,X\rangle\langle P_h,X|\bar{\psi}_j(0)|0\rangle\Big|_{\xi\cdot n_h=\xi_T=0}\end{aligned}$$

where

$$A_T^\alpha(\infty) - A_T^\alpha(-\infty) = \int_{-\infty}^{\infty} d(\eta\cdot P_h) G^{n_h\alpha}(\eta)\Big|_{\eta\cdot n_h=\eta_T=0}.$$

Straightforward Integration





$$\Delta_G(x, x - x_1) \to \int dk^+ d^2k_T dk_1^+ d^2k_{1T} \delta((k - P_\pi)^2 - \mu^2)$$

$$\times Tr\Big[i(\not{k} + m)\gamma_5 i(\not{k} - \not{P}_\pi + \mu)\gamma^\alpha i(\not{k} - \not{k}_1 - \not{P}_\pi + \mu)\gamma_5 i(\not{k} - \not{k}_1 + m) \Big]$$

$$\times \frac{F_1'(k^2, (k - P_\pi)^2)}{k^2 - m^2 + i\epsilon} \frac{F_2'(k_1^2, k \cdot k_1, k^2)}{(k_1^2 - \lambda^2 + i\epsilon)((k - k_1 - P_\pi)^2 - \mu^2 + i\epsilon)} \frac{F_1'((k - k_1)^2, (k - k_1 - P_\pi)^2)}{((k - k_1)^2 - m^2 + i\epsilon)}$$

$$I(x, k^+, k_T; x_1, k_{1T})$$

$$= \frac{1}{(k^2 - m^2 + i\epsilon)} \int \frac{dk_1^+}{(k_1^2 - \lambda^2 + i\epsilon)} \frac{1}{((k - k_1 - P_\pi)^2 - \mu^2 + i\epsilon)} \frac{1}{((k - k_1)^2 - m^2 + i\epsilon)}$$

$$= \frac{1}{4Q^4 x x_1 (x - x_1)(x - x_1 - 1)} \frac{1}{\left(k^+ - \frac{k_T^2 + m^2 - i\epsilon}{\sqrt{2} x Q}\right)}$$

$$\times \int \frac{dk_1^+}{\left(k_1^+ - \frac{k_{1T}^2 + \lambda^2 - i\epsilon}{\sqrt{2} x_1 Q}\right)} \frac{1}{\left(k_1^+ - \left(k^+ + \frac{(k - k_1)_T)^2 + m^2 - i\epsilon}{\sqrt{2}(x_1 - x)Q}\right)\right)} \frac{1}{\left(k_1^+ - \left(k^+ - P_\pi^+ + \frac{(k - k_1)_T^2 + \mu^2 - i\epsilon}{\sqrt{2}(x_1 - (x - 1))Q}\right)\right)}$$

Straightforward Integration

$$\Phi_G(x, x_1 - x) \quad \sim \quad \frac{1}{(k^2 - m^2)} \left\{ \int \frac{dk_1^-}{2\pi\, i} \frac{F_1\left(k_1^-, x, x_1, k_T, k_{1T}\right)}{(k_1^2 - m_1^2 + i\epsilon)((k - k_1)^2 - m^2 + i\epsilon)((P - k + k_1)^2 - M_{s1}^2 + i\epsilon)} \right.$$
$$\left. + \int \frac{dk_1^-}{2\pi\, i} \frac{F_2\left(k_1^-, x, x_1, k_T, k_{1T}\right)}{(k_1^2 - m_1^2 + i\epsilon)((k - k_1)^2 - m^2 + i\epsilon)((P - k_1)^2 - M_{s2}^2 + i\epsilon)} \right\}$$

where, $F_i\left(k_1^-, x, x_1, k_T^2, k_{1T}^2\right)$ **contain numerators and vertex functions**

Parameterize gluon momentum $k_1 = [k_1^-, x_1, k_{1T}],$

- **Assume numerator doesn't grow with** k_1^- **can perform** k_1^- **integrations**

$$\Phi_G \quad \sim \frac{1 - x}{(\mu^2 - k_T^2)} \left\{ \int \frac{dk_1^-}{2\pi i} \frac{F_1(k_1^-, x, x_1, k_T^2, k_{1T}^2)}{(x_1 k_1^- - A_1 + i\epsilon)((x_1 - x)k_1^- - A_2 + i\epsilon)((1 - x + x_1)k_1^- - B_1 + i\epsilon)} \right.$$
$$\left. + \int \frac{dk_1^-}{2\pi i} \frac{F_2(k_1^-, x, x_1, k_T^2, k_{1T}^2)}{(x_1 k_1^- - A_1 + i\epsilon)((x_1 - x)k_1^- - A_2 + i\epsilon)((x_1 - 1)k_1^- - B_2 + i\epsilon)} \right\},$$

Glunonic Pole contribution for Frag. vanishes

Taking the limit $x_1 \to 0$ we get the gluonic pole correlators, for distribution functions ($0 \leq x \leq 1$),

$$\Phi_G(x,x) = -\int d^2k_T d^2k_{1T} \frac{(1-x)F_1(x,0,k_T,k_{1T})\theta(1-x)}{(\mu^2-k_T^2)(xB_1+(1-x)A_2)A_1},$$

and for fragmentation functions ($x = 1/z \geq 1$)

$$\Delta_G(x,x) \quad = \quad 0$$

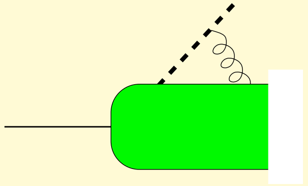
Comments $|h; X\rangle$

$$\Delta_\partial^{\alpha\,[C]}(z) \quad = \quad \tilde{\Delta}_\partial^{\alpha[\not{C}]}\left(\tfrac{1}{z}\right) + C_G^{[U(C)]}\,\pi\Delta_G^{\alpha[\not{C}]}\left(\tfrac{1}{z},\tfrac{1}{z}\right)$$

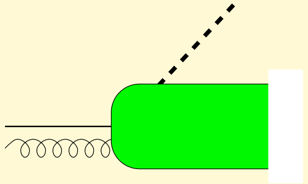
T-odd ***T-odd***

- **All "T-odd" effects for fragmentation in $\tilde{\Delta}_\partial^{\alpha}\left(\frac{1}{z}\right)$ and no "process dependence" $\Delta_G(x,x) = 0$.**

$$\tilde{\Delta}_\partial^{\alpha}\left(\tfrac{1}{z}\right) \quad = \quad \frac{M}{z} i H_1^{\perp(1)}(z)\frac{1}{2}[\not{K},\gamma^\alpha] \neq 0$$

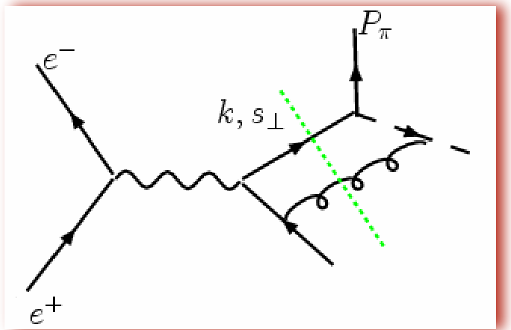


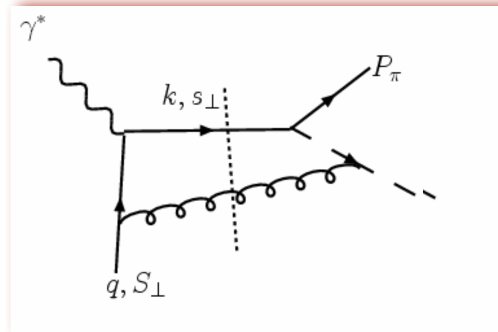
$$\pi\Delta_G\left(\frac{1}{z},\frac{1}{z};K\right) \quad = \quad \frac{M}{z} i \tilde{H}_1^{\perp(1)}(z)\frac{1}{2}[\not{K},\gamma^\alpha] = 0$$



- **Process dependence remains in the T-odd PDFs *"jungle of Wilson lines"* Mulders et al. 2004-present**

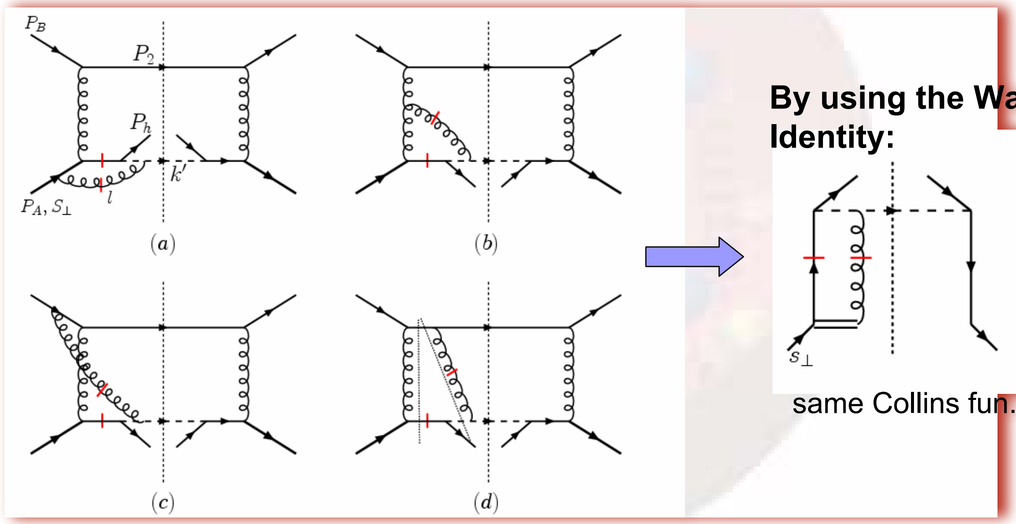
$$C_G^{[\mathcal{U}]}\,\pi\Phi_G^{\alpha}(x,x)$$



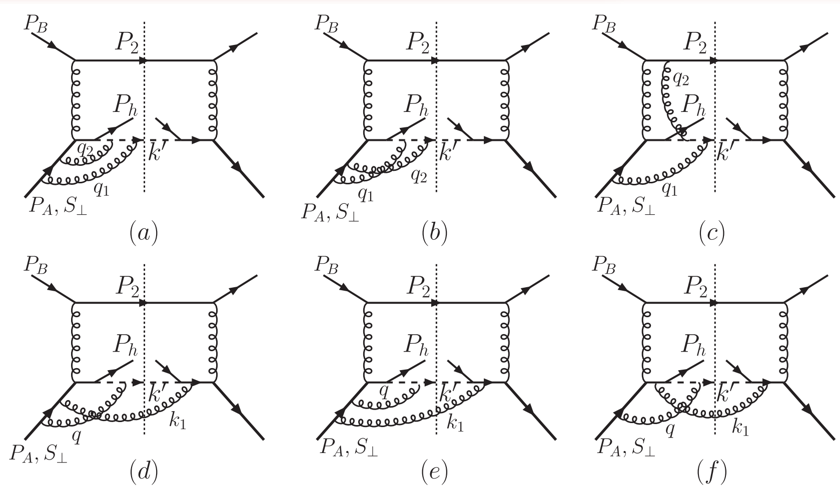


Metz SIDIS& ee plb 2002
Meissner Metz PRL09

Asymmetric Azimuthal Distribution of Hadrons inside a Jet from Hadron-Hadron Collisions



→ ***Yuan prl 2007 pp***

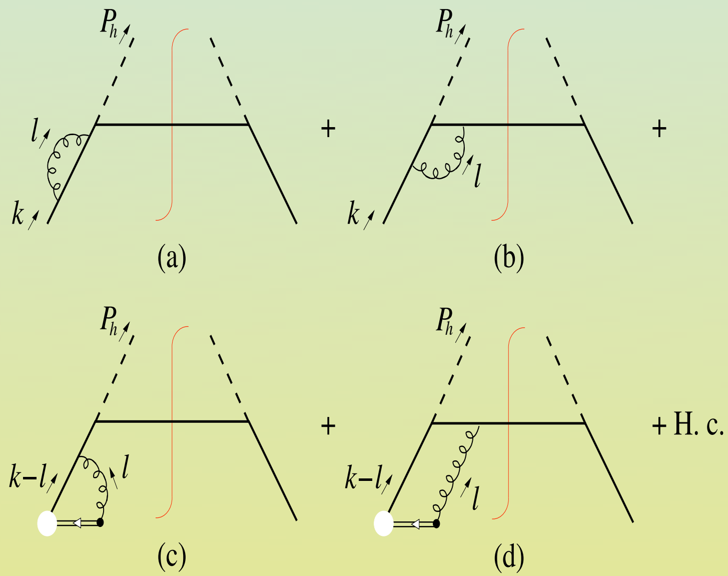


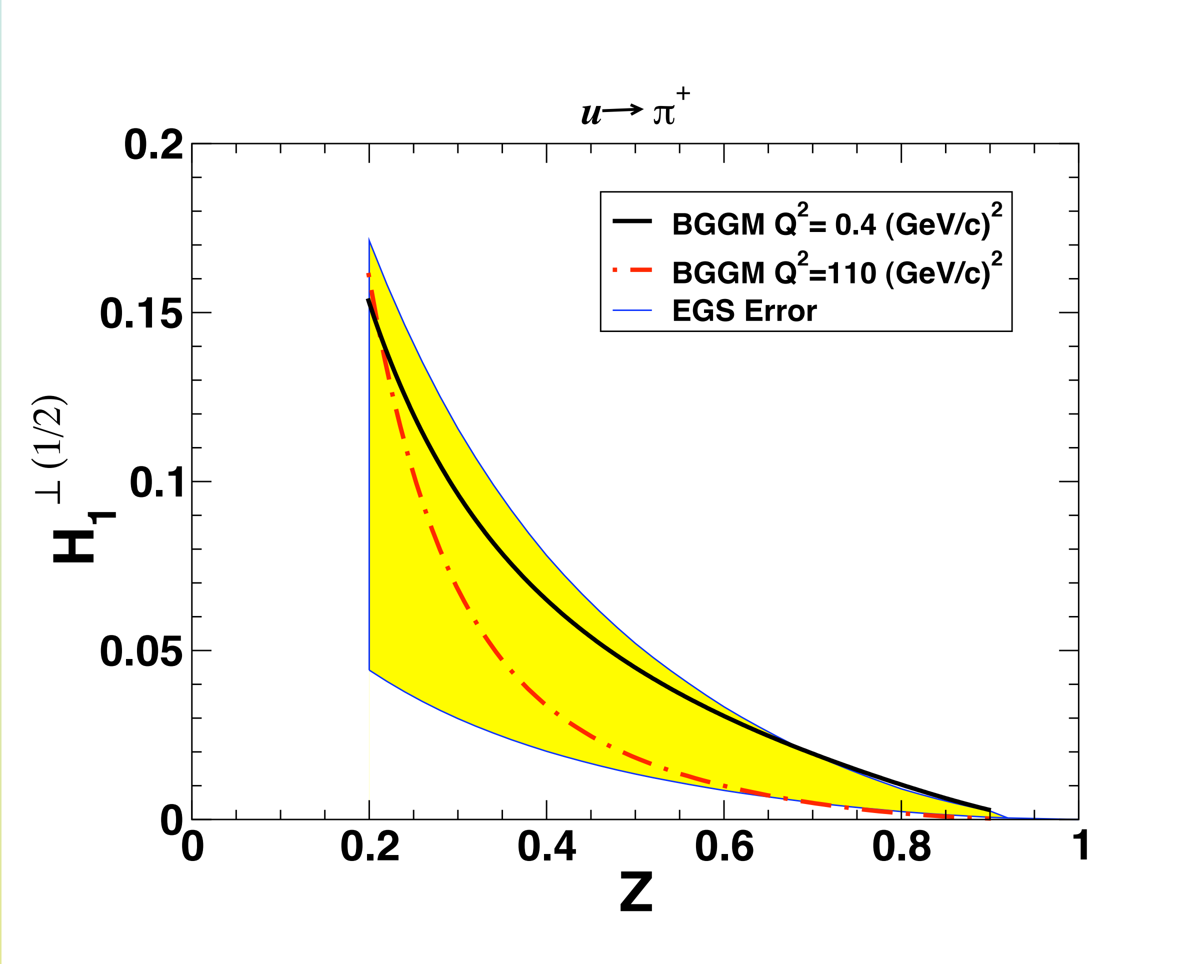
Yuan prd 2008 pp

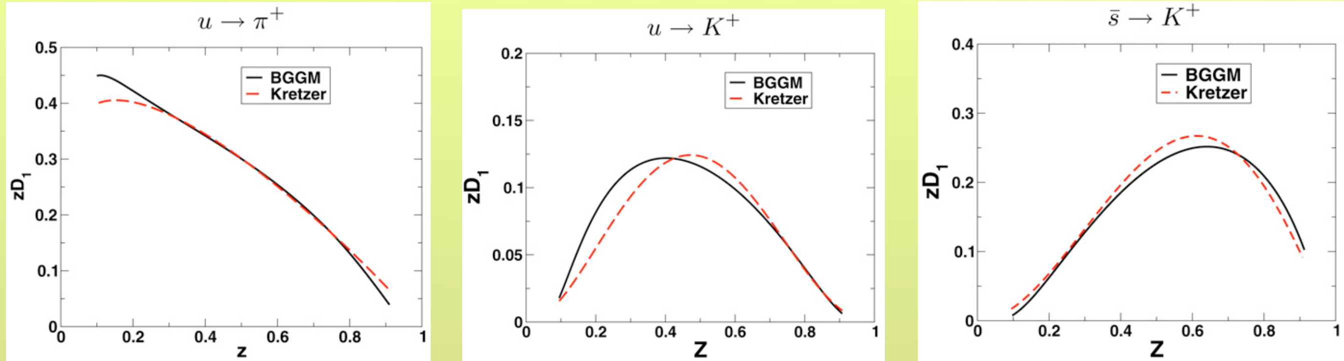
FIG. 6. Example diagrams for two-gluon exchange contributions (a,b,c) and one real gluon radiation contributions (d,e,f).

Bacchetta, L.G., Goldstein, Mukherjee Re-analysis and Kaons

PLB 08

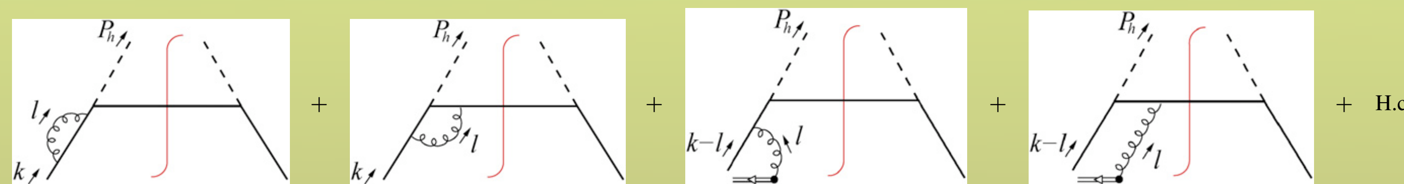






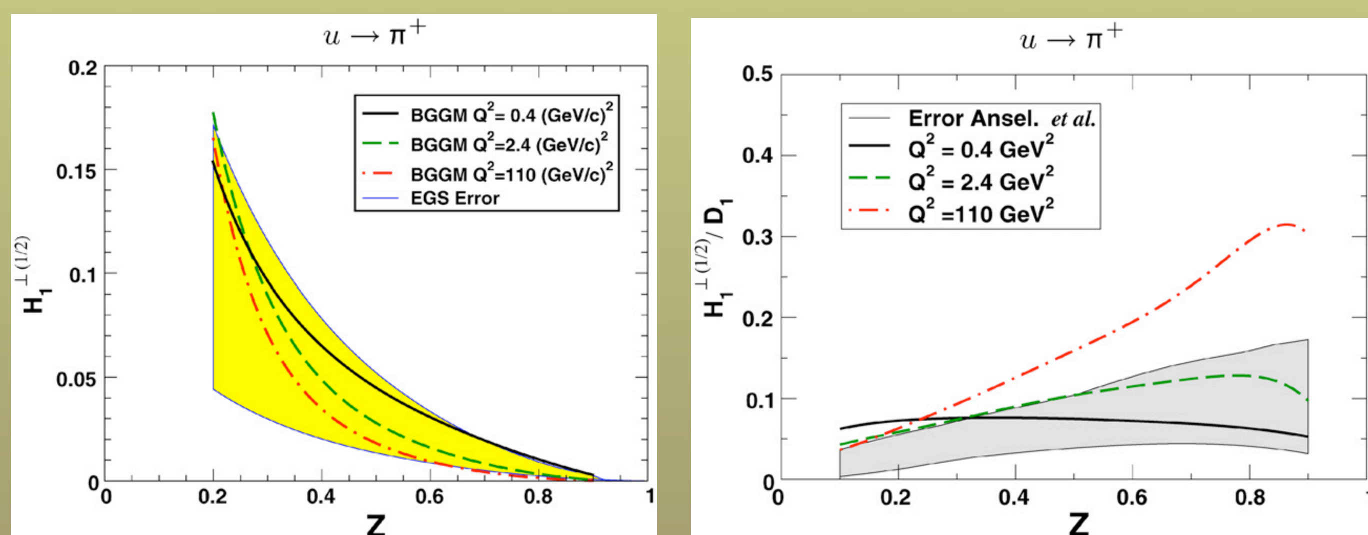
(a) (b) (c)

Fig. 2. Unpolarized fragmentation function $zD_1(z)$ vs. z for the fragmentation (a) $u \to \pi^+$, (b) $u \to K^+$, (c) $\bar{s} \to K^+$ in the spectator model (solid line), with parameters fixed from a fit to the parametrization of [29] (dashed line).



(a) (b) (c) (d)

Fig. 3. Single gluon-loop corrections to the fragmentation of a quark into a pion contributing to the Collins function in the eikonal approximation. “H.c.” stands for the Hermitian conjugate diagrams which are not shown.

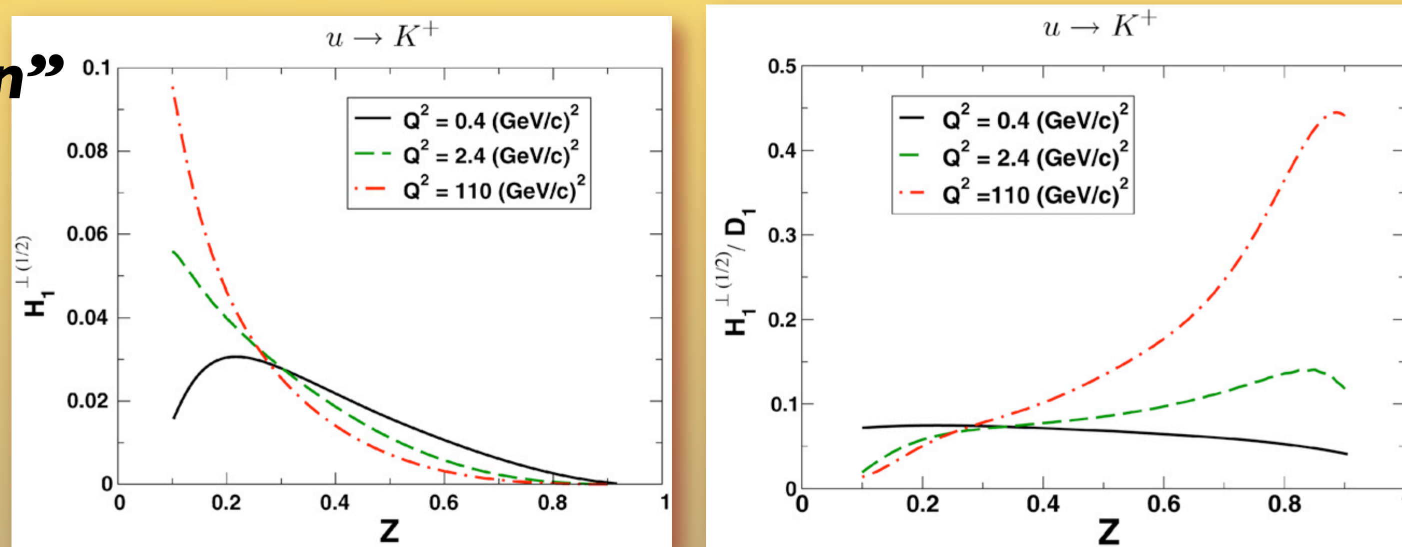


(a) (b)

Fig. 4. Half moment of the Collins function for $u \to \pi^+$ in our model. (a) $H_1^{\perp(1/2)}$ at the model scale (solid line) and at a different scale under the assumption in Eq. (37) (dot-dashed line), compared with the error band from the extraction of Ref. [6], (b) $H_1^{\perp(1/2)}/D_1$ at the model scale (solid line) and at two other scales (dashed and dot-dashed lines) under the assumption in Eq. (38). The error band from the extraction of Ref. [7] is shown for comparison.

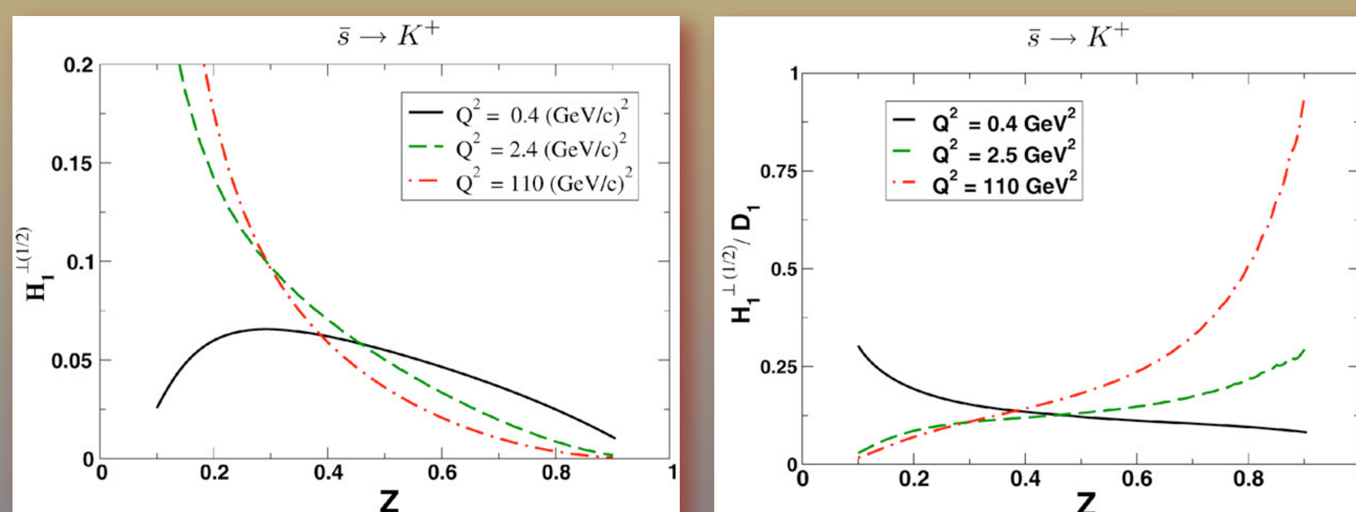
Scaling-"evolution"

$$\left.\frac{H_1^{\perp(1/2)}}{D_1}\right|_{Q_0^2}=\left.\frac{H_1^{\perp(1/2)}}{D_1}\right|_{Q^2},$$



(a) (b)

Fig. 5. Half moment of the Collins function for $u \to K^+$ in our model. (a) $H_1^{\perp(1/2)}$ at the model scale of 0.4 GeV^2, (b) $H_1^{\perp(1/2)}/D_1$ at the model scale (solid line) and at two other scales (dashed and dot-dashed lines) under the assumption in Eq. (38).



(a) (b)

Fig. 6. Half moment of the Collins function for $\bar{s} \to K^+$ in our model. (a) $H_1^{\perp(1/2)}$ at the model scale of 0.4 GeV^2, (b) $H_1^{\perp(1/2)}/D_1$ at the model scale (solid line) and at two other scales (dashed and dot-dashed lines) under the assumption in Eq. (38).

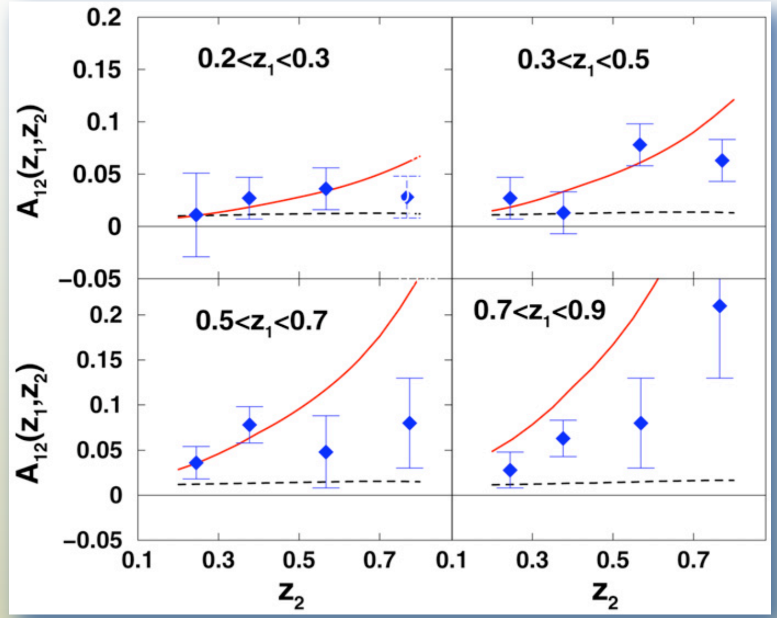


Fig. 7. Azimuthal asymmetry $A_{12}(z_1, z_2)$ for the production of two pions as a function of z_2 and integrated in bins of z_1 at $Q^2 = 110.7$ GeV^2. Dashed lines are obtained assuming Eq. (37), solid lines assuming Eq. (38). Note that the last z_1 bin in our calculation is narrower than in the corresponding experimental measurement.

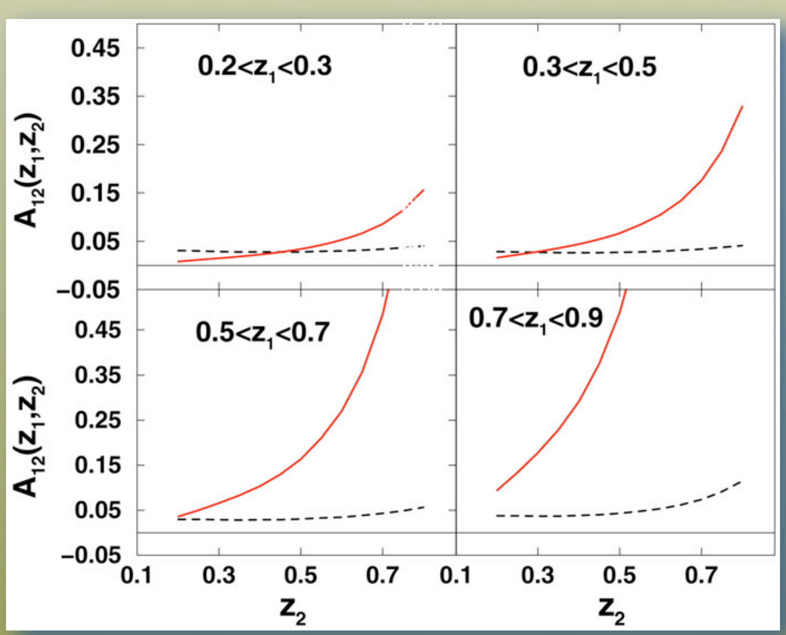
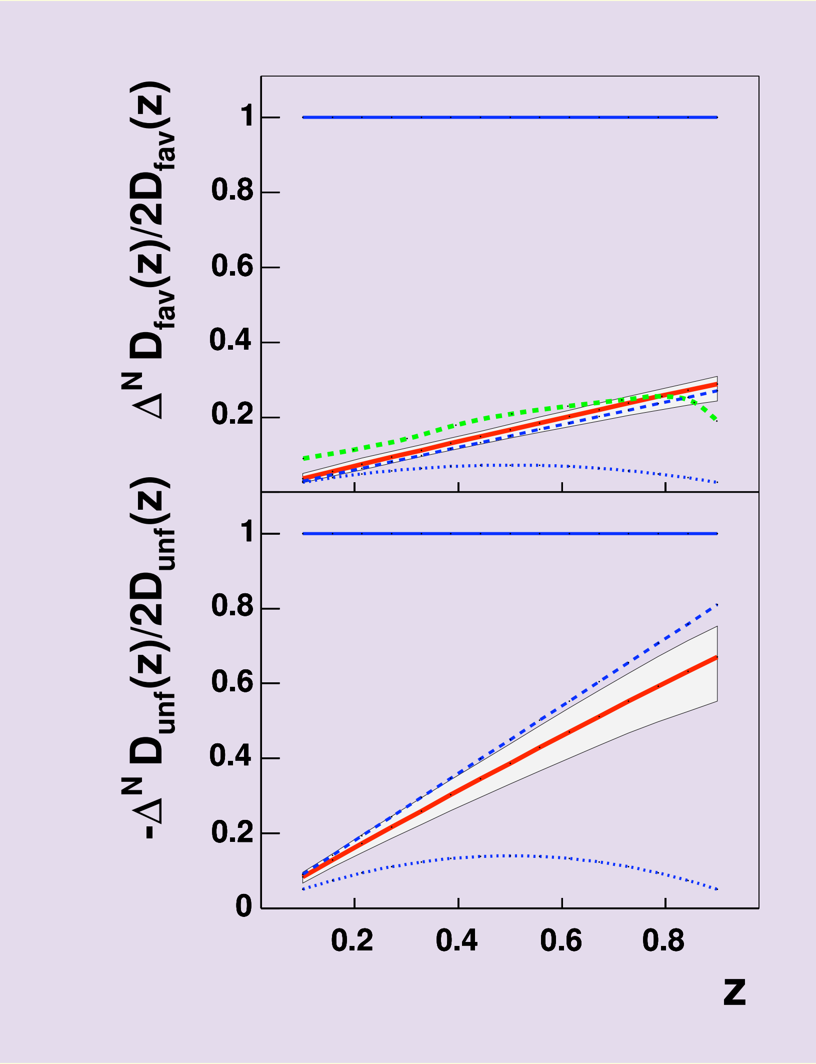


Fig. 8. Azimuthal asymmetry $A_{12}(z_1, z_2)$ for the production of two kaons as a function of z_2 and integrated in bins of z_1 at $Q^2 = 110.7$ GeV^2. Dashed lines are obtained assuming Eq. (37), solid lines assuming Eq. (38).

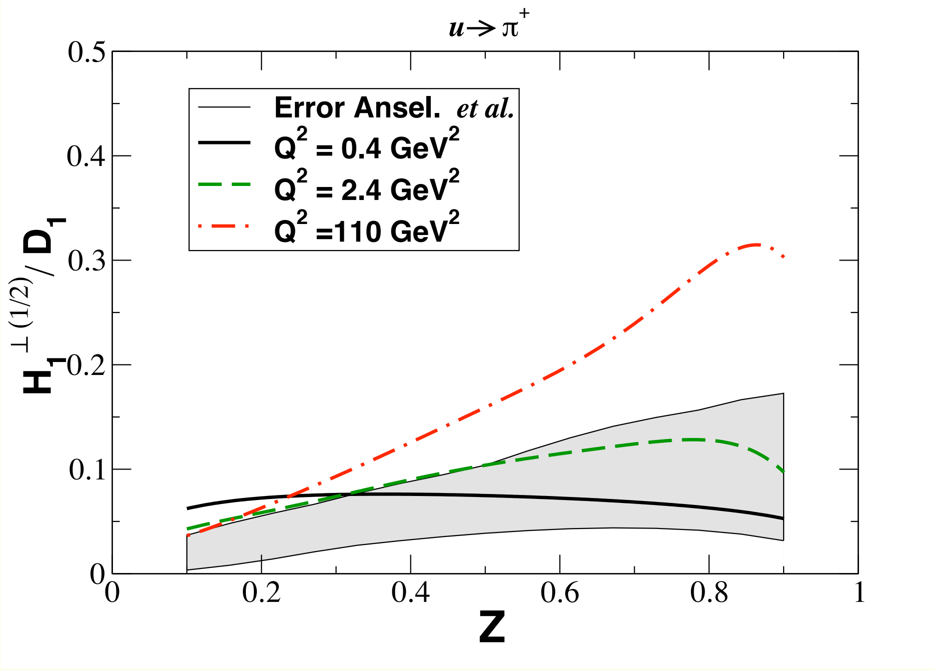
Anselmino Prokudin et al. Ferrara Transversity 08



compared to Ref. [1] (dashed line), Ref. [2] (dotted line), and Ref. [3] (dashed green line)

[1] A. V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. **D73**, 094025 (2006).
[2] W. Vogelsang and F. Yuan, Phys. Rev. **D72**, 054028 (2005).
[3] A. Bacchetta, L. Gamberg, G. R. Goldstein, A. Mukherjee PLB659:234-243,2008.

Bacchetta, Gamberg, Goldstein, Mukherjee PLB 08



Reliability of Transversity Extraction Universality of Collins Fragmentation Function

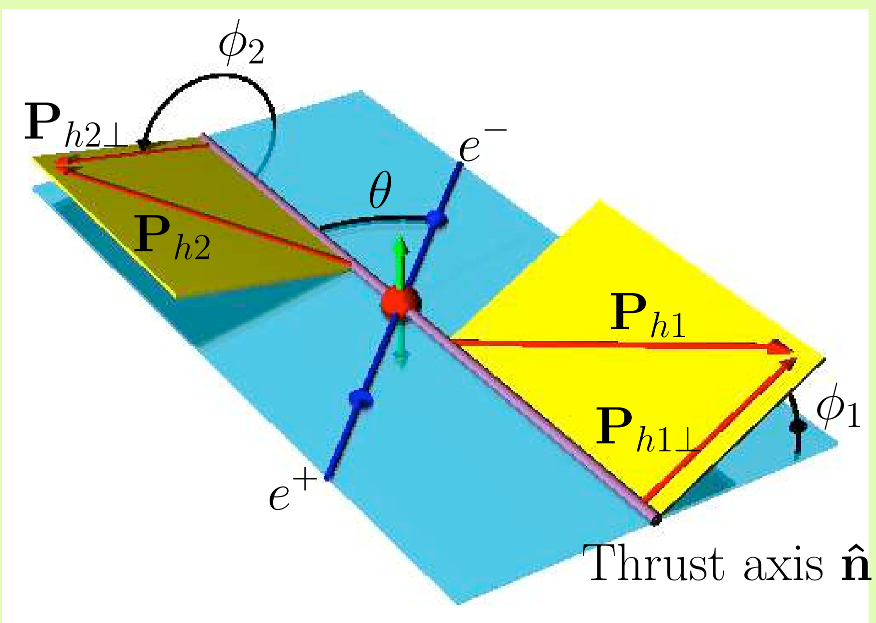
Belle KEKB measurement of the Collins Frag. Function PRL 2006 *&* arXiv:0805.2975

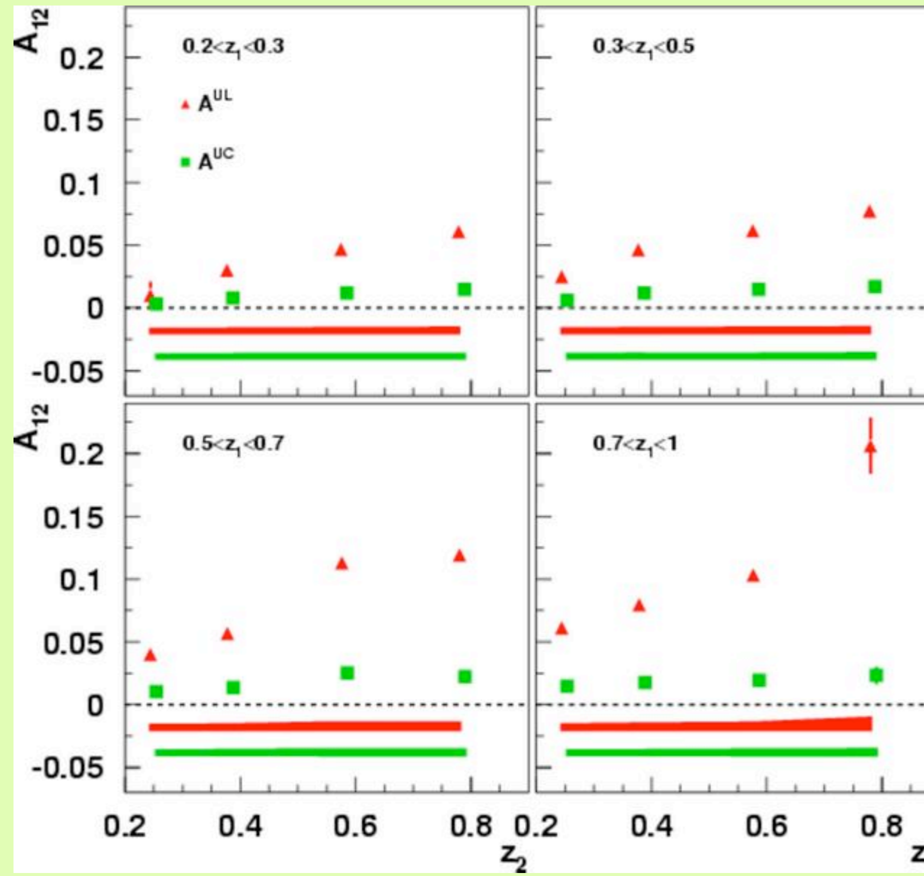
$$A_{12}(z_1, z_2) = \frac{\langle \sin^2\theta \rangle}{\langle 1+\cos^2\theta \rangle}(P_U - P_L),$$

where

$$P_U = \frac{\sum_q e_q^2 (H^{\perp(1/2)}_{1(q\to\pi^+)}(z_1) H^{\perp(1/2)}_{1(\bar{q}\to\pi^-)}(z_2) + H^{\perp(1/2)}_{1(q\to\pi^-)}(z_1) H^{\perp(1/2)}_{1(\bar{q}\to\pi^+)}(z_2))}{\sum_q e_q^2 (D_{1(q\to\pi^+)}(z_1) D_{1(\bar{q}\to\pi^-)}(z_2) + D_{1(q\to\pi^-)}(z_1) D_{1(\bar{q}\to\pi^+)}(z_2))},$$

$$P_L = \frac{\sum_q e_q^2 (H^{\perp(1/2)}_{1(q\to\pi^+)}(z_1) H^{\perp(1/2)}_{1(\bar{q}\to\pi^+)}(z_2) + H^{\perp(1/2)}_{1(q\to\pi^-)}(z_1) H^{\perp(1/2)}_{1(\bar{q}\to\pi^-)}(z_2))}{\sum_q e_q^2 (D_{1(q\to\pi^+)}(z_1) D_{1(\bar{q}\to\pi^+)}(z_2) + D_{1(q\to\pi^-)}(z_1) D_{1(\bar{q}\to\pi^-)}(z_2))}.$$

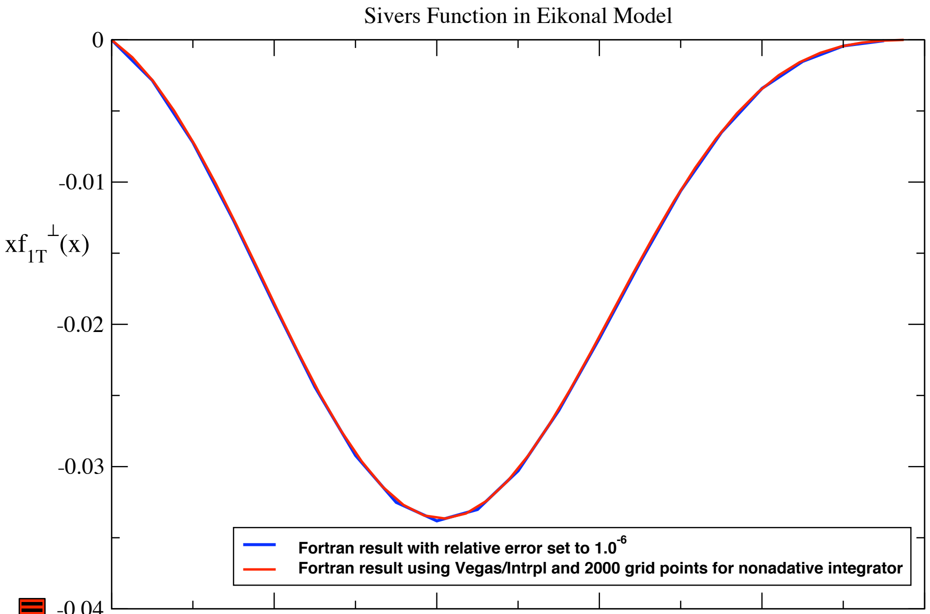


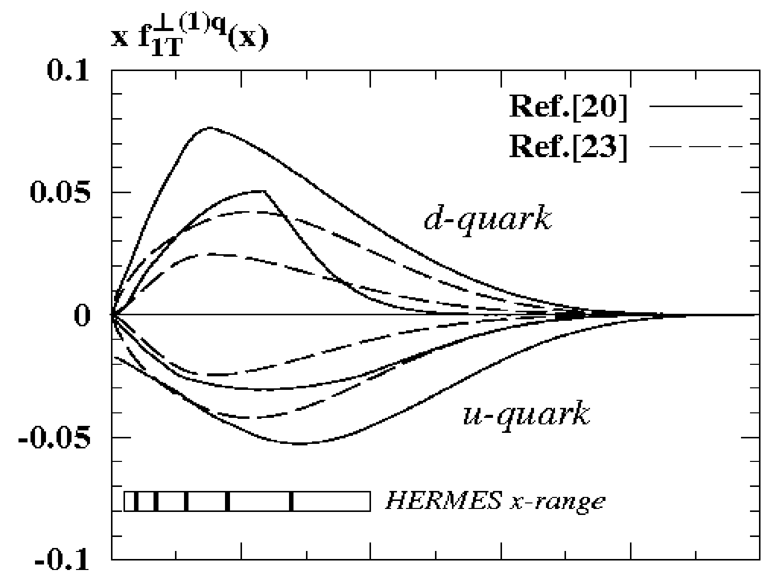


Ralf Seidl EIC Workshop, Hampton, VA May 08

Preliminary Results

$$f_{1T}^{\perp,(1)u}(x) = -\frac{1}{2(1-x)M^2}\int\frac{d^2p_T}{(2\pi)^2}\,p_T^y\,I^y(x,|\vec{p}_T|)\,E^u\left(x,0,-\frac{\vec{p}_T^{\,2}}{(1-x)^2}\right)$$





- Lensing Function behaves differently compared to the one-gluon exch.

Outlook: Possible "improvements" of the model:

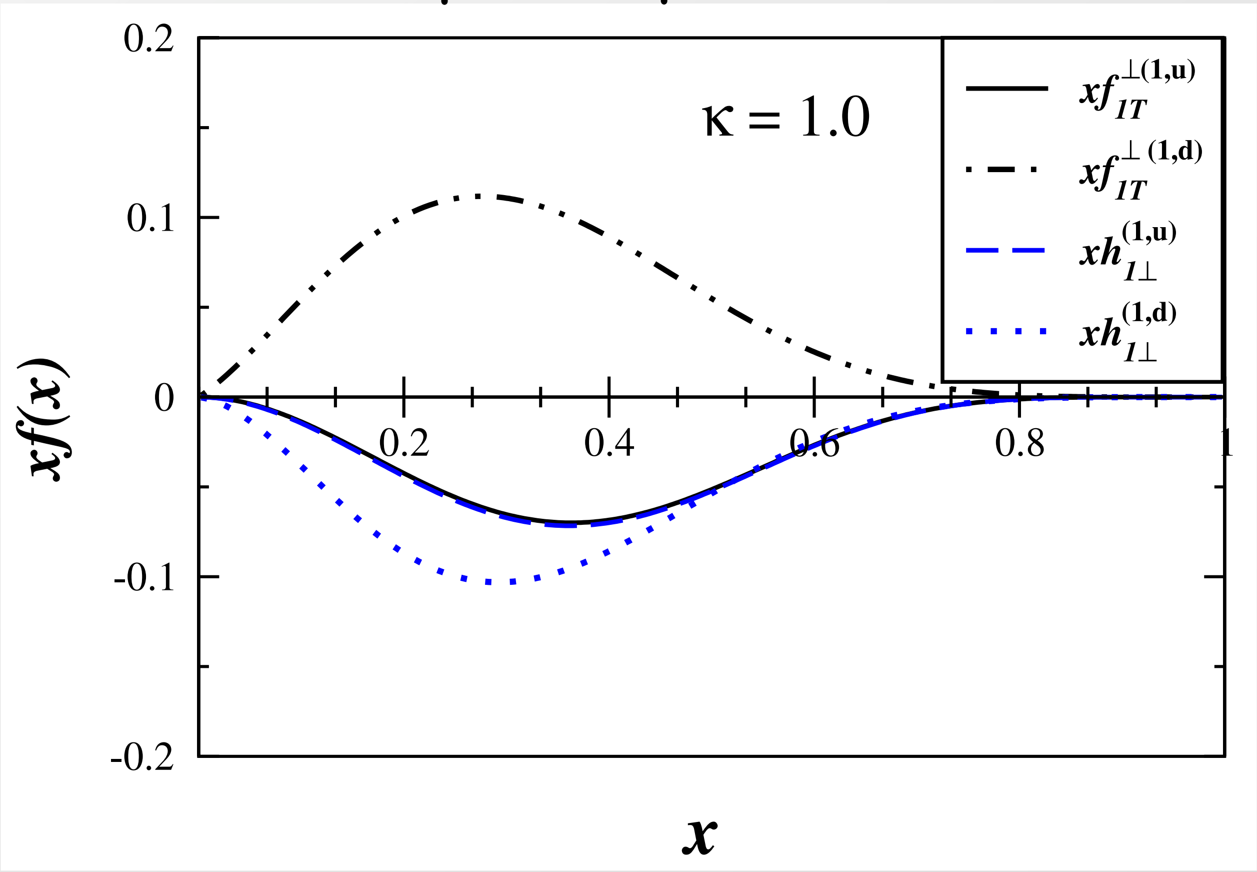
- Implementation of non-perturbative scalar-glue/ fermion-glue vertices.
- Inclusion of axial-vector diquarks → prediction for d-quarks
- Try other nucleon-quark-diquark vertices.

$\langle\cos(2\phi_h)\rangle$: Model 1

Gamberg et al.

L. P. Gamberg et al., Phys Rev D67:071504, 2003
L. P. Gamberg and G. R. Goldstein, arXiv:0708.0324, 2007

Same sign u and d Boer-Mulders function
from a diquark spectator model

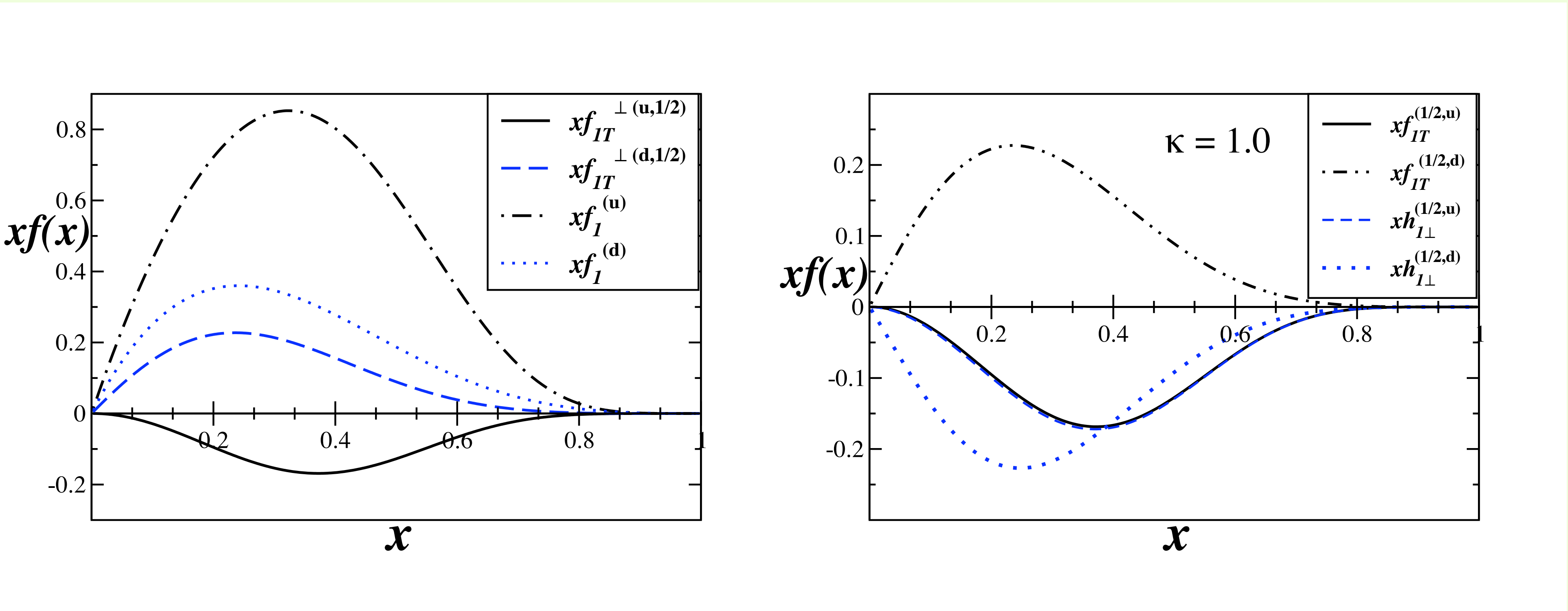


Collins calculated in the spectator framework
A. Bacchetta, et al., Phys. Lett. B 659, 234 (2008).

Flavor Dependence: Results & Phenomenology

Flavor-dependent PDFs from diquark models: $u = \frac{3}{2}s + \frac{1}{2}a$, $d = a$,

moments: $h_1^{\perp(1/2)}(x) = \int d^2\vec{p}_T \frac{|\vec{p}_T|}{M} h_1^{\perp}(x, \vec{p}_T^{\,2})$ L.G. Goldstein, Schlegel PRD 2008



- Comparison to $f_1^{(u,d)}$ (Glück, Reya, Vogt) $\rightarrow$ parameters of the model, e.g. diquark masses, normalization...
- Comparison to parameterization of Sivers function $f_{1T}^{\perp}$ $\rightarrow$ size and sign of FSI Anselmino et al. 2005 PRD
- Boer Mulders up and down are negative is spectator model and $f_{1T}^{(u)} \sim h_1^{\perp(u)}$

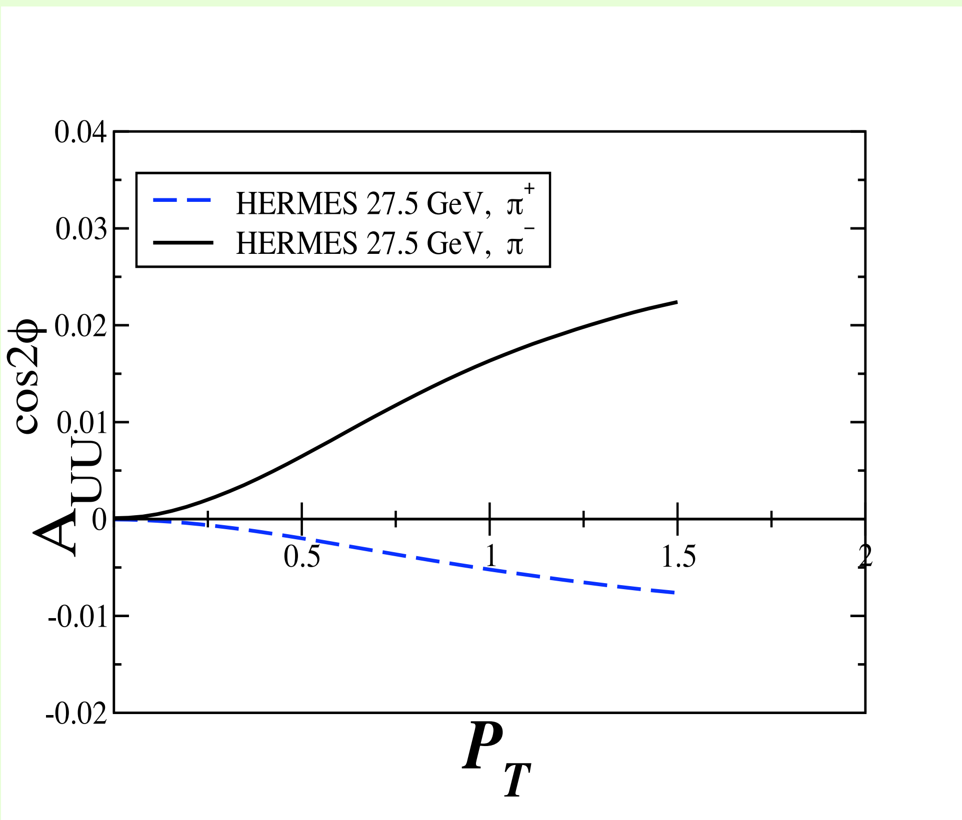
HERMES & CLAS12 PAC 30-Avakian, Meziani. . . L.G. . . L.G. Goldstein Schlegel PRD 2008

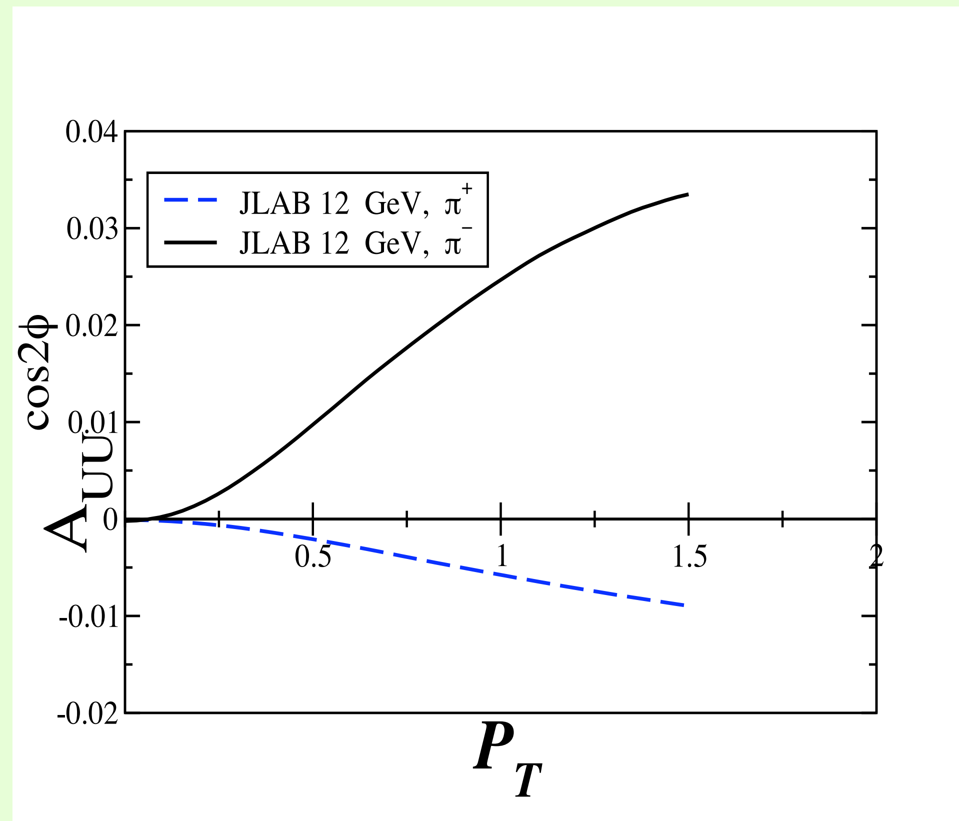
$$A_{UU}^{\cos(2\phi_h)} \propto \sum_a e_a^2 \int d^2p_T d^2k_T\, \delta^{(2)}\left(\vec{p}_T - \vec{k}_T - \frac{\vec{P}_{h\perp}}{z_h}\right) \frac{\vec{k}_T\cdot\vec{p}_T - 2(\vec{h}\cdot\vec{k}_T)(\vec{h}\cdot\vec{p}_T)}{M m_\pi} h_1^{\perp(a)} H_1^{\perp(a)}$$

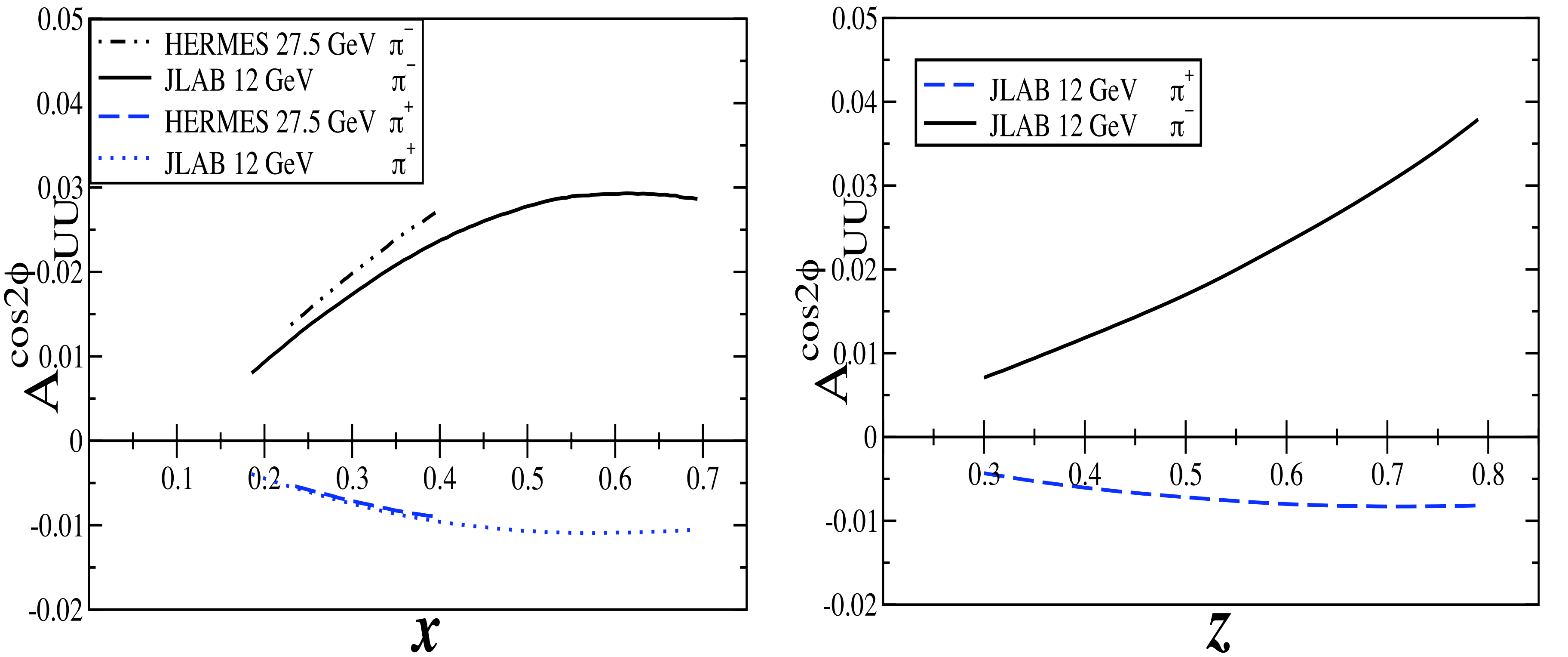
Model assumption:

Dis-favored fragmentation

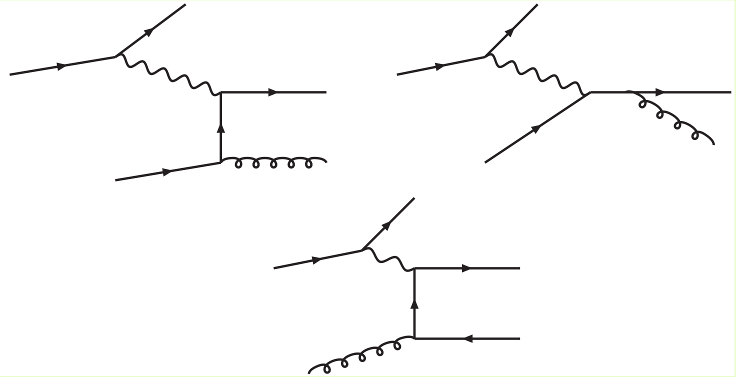
$H_1^{\perp\ (d\to\pi^+)} = -H_1^{\perp\ (u\to\pi^+)}$, $\quad \sum_h \int_0^1 dz H^{\perp(1)}_{1(q\to h)} = 0$ **Schäfer and Teryaev, PRD 2000**







- **Theory issues gluon brem and Cahn effect see Barone, Prokudin and Ma arXiv:0804.3024 [hep-ph] Bacchetta, Boer, Diehl, Mulders arXiv:0803.0227**



<cos(2φh)>: Model 1

Gamberg et al.

L. P. Gamberg et al., Phys Rev D67:071504, 2003
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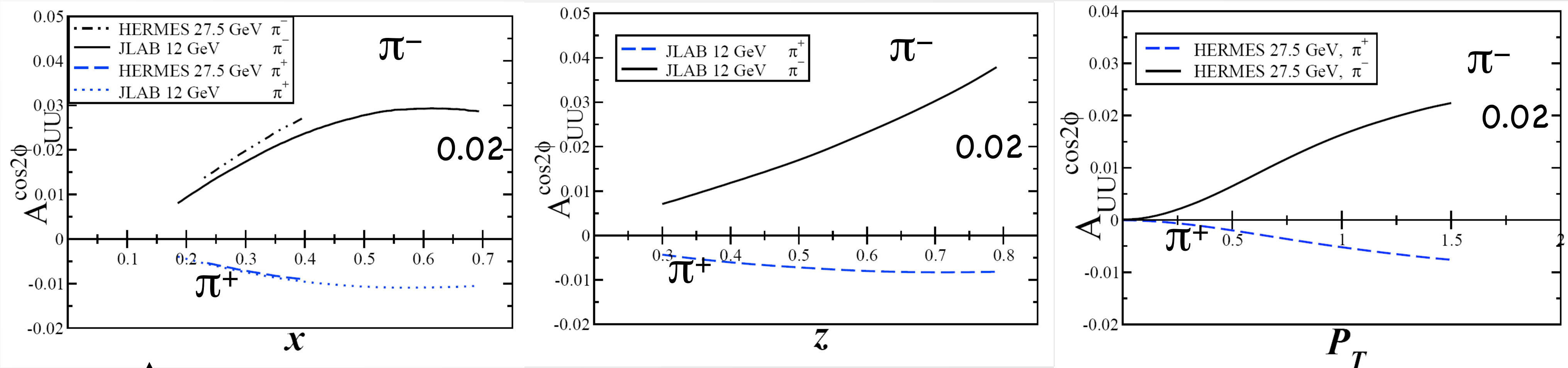
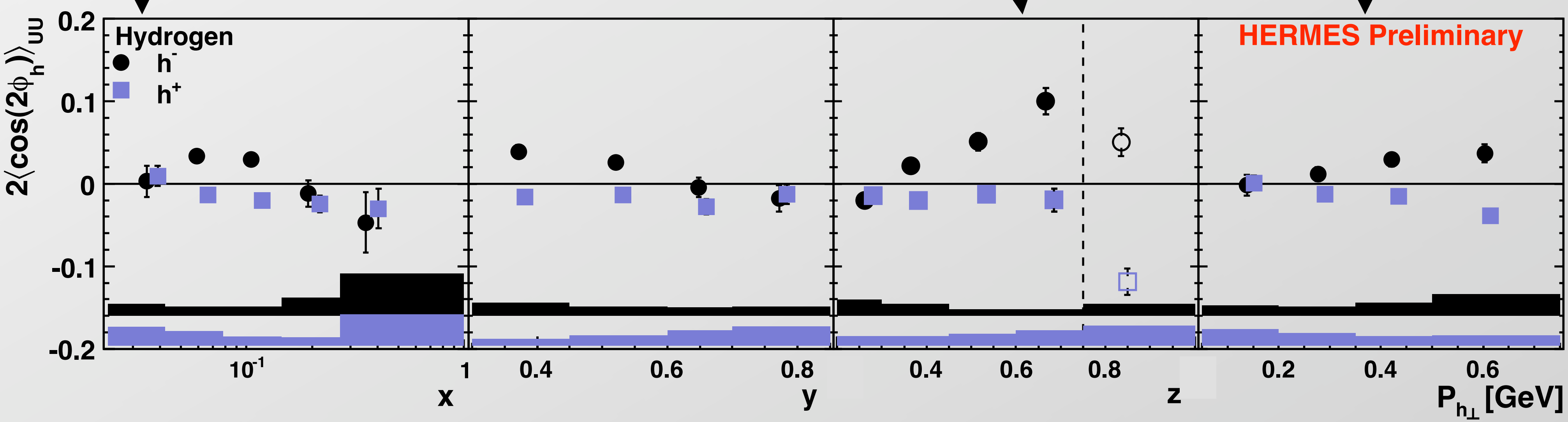
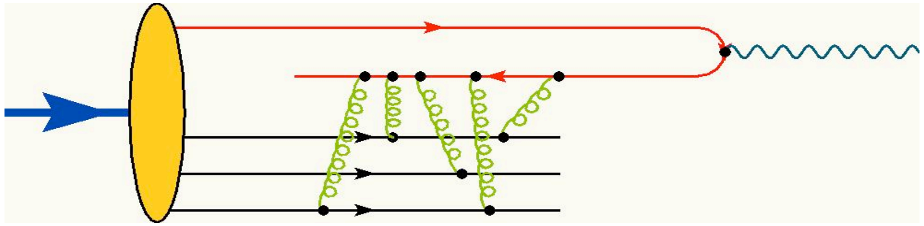
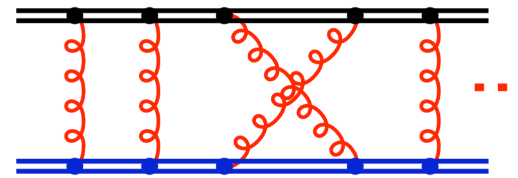


Diquark spectator model does well… without Cahn term

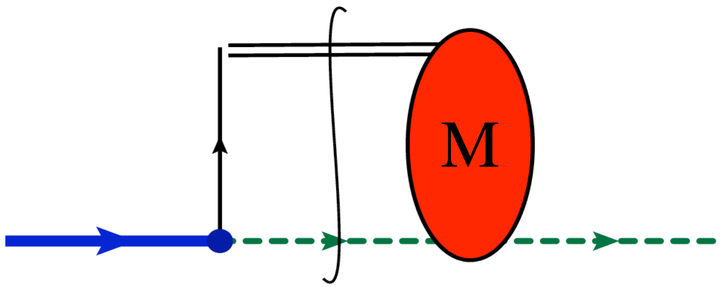






Can we do better? Can we learn about the quality of the relations?

[L. Gamberg, M.S., in preparation]



- Still work within spectator framework, but *non-perturbative model of FSI*.
- In order to separate out GPDs, "cut" the diagram → "natural" picture of FSI.

$$f_{1T}^{\perp,(1)u}(x) = -\frac{1}{2(1-x)M^2}\int \frac{d^2p_T}{(2\pi)^2}\, p_T^y I^y(x,|\vec{p}_T|) E^u\left(x,0,-\frac{\vec{p}_T^{\,2}}{(1-x)^2}\right)$$

Lensing function given in terms of quark-diquark scattering amplitude M:

$$I^y(x,\vec{p}_T) = p_T^y \Im[M(x,|\vec{p}|_T)] + \int \frac{d^2q_T}{(2\pi)^2}\,(2q_T^y - p_T^y)\Im[M(x,|\vec{q}_T|)]\Re[M(x,|\vec{q}_T - \vec{p}_T|)]$$

Summary.... Improvements FSI times Spatial Distortion

Conjecture: factorization of final state interactions and spatial distortion:

$$\boxed{\langle k_T^i\rangle = -M\epsilon_T^{ij}S_T^j\, f_{1T}^{\perp,(1)}(x) \simeq \int d^2b_T\,\mathcal{I}^i(x,\vec{b}_T)\,\frac{\vec{b}_T\times\vec{S}_T}{M}\,\frac{\partial}{\partial b_T^2}\mathcal{E}(x,\vec{b}_T^2)}$$

$\mathcal{I}^i(x,\vec{b}_T^2)$: Lensing Function = net transverse momentum

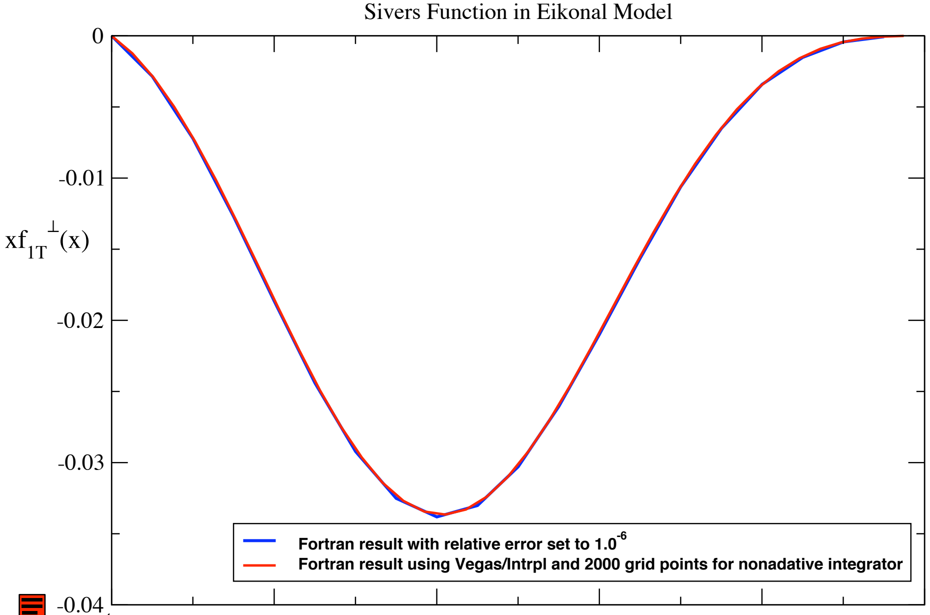
- Av. transv. momentum of transv. pol. partons in an unpol. hadron:

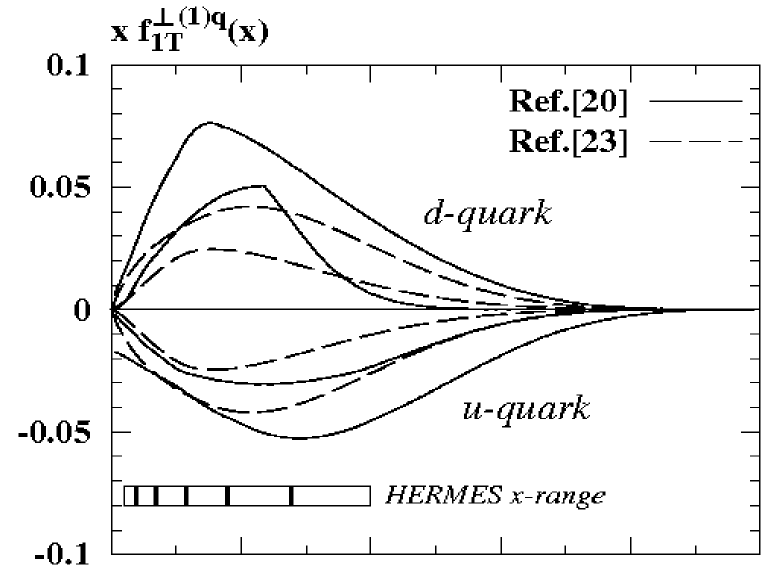
$$\boxed{\langle k_T^i\rangle^j(x) = \int d^2k_T\,k_T^i\,\frac{1}{2}\left(\Phi^{[i\sigma^{i+}\gamma^5]}(S)+\Phi^{[i\sigma^{i+}\gamma^5]}(-S)\right)}$$

$$\Rightarrow\quad \boxed{-2M^2\,h_1^{\perp,(1)}(x) \simeq \int d^2b_T\,\vec{b}_T\cdot\vec{\mathcal{I}}(x,\vec{b}_T)\,\frac{\partial}{\partial b_T^2}\left(\mathcal{E}_T+2\tilde{\mathcal{H}}_T\right)(x,\vec{b}_T^2)}$$

Preliminary Results

$$\boxed{f_{1T}^{\perp,(1)u}(x) = -\frac{1}{2(1-x)M^2}\int \frac{d^2p_T}{(2\pi)^2}\, p_T^y I^y(x,|\vec{p}_T|) E^u(x,0,-\frac{\vec{p}_T^{\,2}}{(1-x)^2})}$$





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